Uniqueness and stability properties of multiphase mean curvature flow: An approach based on the variational (gradient flow) structure of the problem

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For many evolution problems for interfaces - like for instance multiphase mean curvature flow or the Mullins-Sekerka equation - appropriate weak solutions are known to exist globally in time, but the uniqueness of such weak solutions is either unknown or even known to fail. At the same time, due to geometric singularities strong solution concepts are in general limited to local in time existence results. In the absence of a comparison principle, the relation between weak and strong solutions for interface evolution problems has remained a mostly open question.

We establish a weak-strong uniqueness principle for planar multiphase mean curvature flow: We prove that for mean curvature flow of planar networks, there is only a single BV solution prior to the first topology change. Our approach relies on the variational structure of mean curvature flow, being the gradient flow of the interface energy functional. For many minimization problems for interface energy functionals, the method of calibrations has made it possible to deduce the uniqueness of minimizers. Our approach to weak-strong uniqueness for mean curvature flow relies on a novel gradient-flow analogue of this concept of calibrations, basically allowing us to show that the route of steepest descent in the energy landscape is unique and stable with respect to perturbations.

In the last part of the talk, we will discuss possible future applications of our concept, including the quantitative convergence of phase-field approximations for mean curvature flow.