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Title of the talk: Four Critical Numbers for Elliptic Systems with Block Structure

Abstract: We will discuss operators of the form $Lu = a^{-1}\mathrm{div}_x(d\nabla_x u)$ on $\mathbb{R}^n$, where a, d are bounded and accretive functions. Such multiplicative perturbations of divergence form operators with complex coefficients naturally arise as the boundary operator for the elliptic system

$$(^*) \qquad \partial_t(a\partial_t u) + \mathrm{div}_x(d\nabla_x u) = 0$$

in block form in the upper half-space described by $t > 0, x \in \mathbb{R}^n$.

We explore the limitations of operator theory and harmonic analysis for L in Lebesgue, Hardy and Sobolev spaces by asking questions of the following type. What is the largest set of exponents p such that on L^p the (i) Poisson semigroup $(e^{-t\sqrt{L}})_{t\geq 0}$ is bounded, (ii) L has a bounded H^∞-calculus, (iii) the Dirichlet problem for the system $(^*)$ is well-posed, (iv) the Riesz transform $\nabla_x L^{-1/2}$ is bounded? Four critical numbers keep appearing as the answer to all these (and many more) questions. The same numbers also encode the answer to questions of a seemingly very different type: For instance, certain critical numbers being strictly below $p = 1$ is equivalent to having Gaussian kernel bounds for the resolvents of L.

My talk is based on a recent monograph jointly written with Pascal Auscher.