

Well-posedness and optimal velocity control of a Brinkman–Cahn–Hilliard system with curvature effects

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Diffuse-interface models for multiphase flows have attracted considerable interest due to their ability to describe complex interfacial dynamics, including curvature effects, within a unified and energetically consistent framework. In this talk, we present joint results for a Brinkman–Cahn–Hilliard system coupling a sixth-order phase-field evolution with a Brinkman-type momentum equation with variable shear viscosity. The Cahn–Hilliard equation includes a nonconservative source term modeling mass exchange, while the momentum equation involves a forcing term that is not divergence-free. We prove the existence of weak solutions in a divergence-free variational framework and, in the case of constant mobility and shear viscosity, establish uniqueness and continuous dependence on the forcing. We also analyze the Darcy limit and obtain existence results for the corresponding reduced system. Finally, we consider an optimal control problem with distributed velocity control, prove the existence of optimal controls, show the Fréchet differentiability of the control-to-state operator, and derive first-order necessary optimality conditions via an adjoint system.