On the Self-Propelled Motion of a Rigid Body in a Viscous Liquid by Time-Periodic Boundary Data

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Consider a rigid body, $\mathcal{B}$, constrained to move by translational motion in an unbounded viscous liquid. The driving mechanism is a given distribution of time-periodic velocity field, $\mathbf{v}_*$, at the interface body-liquid, of magnitude δ (in appropriate function class). The main objective is to find conditions on $\mathbf{v}_*$ ensuring that $\mathcal{B}$ performs a non-zero net motion, namely, $\mathcal{B}$ can cover any given distance in a finite time. This study is mostly motivated by the self-propulsion of vibrating bodies in viscous liquids, where the vibration is generated by the oscillation of an internal mass. We show that the approach to the problem depends on whether the averaged value of $\mathbf{v}_*$ over a period of time is (case (b)) or is not (case (a)) identically zero. In case (a) we solve the problem in a relatively straightforward way, by showing that, for small δ, it reduces to the study of a suitable and well-investigated time-independent Stokes (linear) problem. In case (b), however, the question is much more complicated, because we show that it *cannot* be brought to the study of a linear problem. Therefore, in case (b), self-propulsion is a genuinely nonlinear issue that we solve directly on the nonlinear system by a contradiction argument. In this way, we are able to give, also in case (b), sufficient conditions for self-propulsion (for small δ). Finally, we demonstrate, by means of counterexamples, that such conditions are, in general, also necessary.