Project E5

Statistical and numerical methods in modeling of financial derivatives and valuation of risk

DFG Research Center MATHEON
mathematics for key technologies
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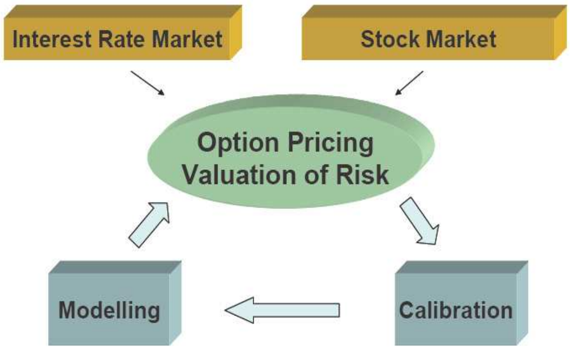
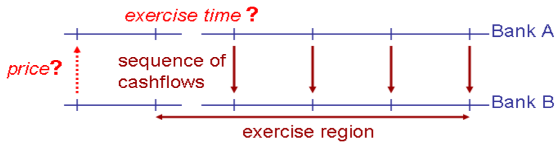
Domain of Expertise: Simulation and Calibration

Main problems and goals



Complex structured callable instruments



Callable interest rate products lead to high dimensional optimal stopping problems:

- Iterative methods for the exercise policy: Kolodko and Schoenmakers, *Fin. Stoch.* (2006); Bender and Schoenmakers, *Adv. in Appl. Prob.* (2006)
- Speed up by scenario selection methods: Bender, Kolodko, Schoenmakers, *Quant. Fin.* (2008)

True upper bounds by linear Monte Carlo

A linear Monte Carlo method for dual upper estimates:

$$\widehat{V}^{up} = \frac{1}{N}\sum_{n=1}^{N} \max_{0\le j\le k}\left[{}_nZ_j - \sum_{t_i\in\pi;\,0\le t_i<T_j} \widehat{h}^{\pi}(t_i, {}_nL_j)\cdot \Delta_n^{\pi} W_i\right]$$

The functions $h^{\pi}(t_i, L)$ are obtained via a regression estimator of a discretized Clark-Ocone derivative at a fine grid $\pi = \{t_i\} \subset \{T_j\}$.
Belomestny, Bender, Schoenmakers, *Math. Fin.* (2008);
Belomestny, Milstein, Spokoiny, *Quant. Fin.* (2009)

Efficient sensitivities by Monte Carlo

Popular but naive Monte Carlo estimators may suffer from exploding variance. A new estimator resolves this problem:

$$\frac{\widehat{\partial I}}{\partial x}(x) = \frac{1}{M}\sum_{m=1}^{M}\frac{\partial}{\partial x}\frac{p(x,g(x,{}_m\xi))u(g(x,{}_m\xi))}{\phi(x,g(x,{}_m\xi))}$$

where $g(x,\xi)$ is a proxy sampler with known density ϕ. Fries, Kampen, *J. Comp. Fin.* (2007); Kampen, Kolodko, Schoenmakers, *SIAM J. Sc. Comp.* (2008)

Realistic modeling of Libor rates

Libor model with jumps and stochastic volatility:

$$\frac{dL_i(t)}{L_i(t)} = \underbrace{\Gamma_i^\top dW^{(i+1)}(t)}_{\text{Continuous Part}} + \underbrace{\int_E \psi_i(t,u)\left(\mu-\nu^{(i+1)}\right)(dt,du)}_{\text{Jump Part}}$$

Belomestny, Schoenmakers, *Quant. Fin.* (to appear)
Future goal:
Multi-factor Jump-Libor modeling by infinite activity Lévy measures and Lévy copulas. Papapantoleon, Schoenmakers (in preparation)

Modeling by affine processes and beyond

Functional series expansions for the characteristic function $p = Ee^{iu^\top X_s^{0,x}}$ of a multi-dimensional affine process:

$$\ln p(s,x,u) = \ln\left[\sum_{r\ge0} h_{r,0}(u)(1-e^{-\eta s})^r\right] + x^\top \frac{\sum_{r\ge1} h_r(u)(1-e^{-\eta s})^r}{\sum_{r\ge0} h_{r,0}(u)(1-e^{-\eta s})^r}$$

All ingredients can be obtained from the affine generator.
Belomestny, Kampen, Schoenmakers: *J. Funct. Anal.* (2009)
Future goal:
Generalization to other processes (with E10)

Optimal control in finance

New regression methods and convergence analysis for the optimal control problem

$$Y_r^* = \sup_{\mathbf{a}\in\mathcal{A}_r,\,\tau\in\mathfrak{T}_r} \mathrm{E}^{\mathbf{a}}\left[\sum_{s=r}^{\tau-1} f_s(X_s,a_s) + g_\tau(X_\tau)\,\middle|\,\mathcal{F}_r\right]$$

Belomestny, Kolodko, Schoenmakers, *SIAM J. Contr. Opt.* (to appear)

Future goals in financial optimization

Problems in illiquid markets, models with transaction costs, large investors etc.

Optimal stopping and control for utility functionals (with E2)

Dimension reduction, nonstationary time series analysis (with A3/F10)

Dual methods for Lévy processes (with E9), multiple stopping, and energy options

Industrial contracts

LBB LandesBank Berlin
HSH NORDBANK
WGZ BANK Die Initiativbank