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Nonlocal and nonlinear effects in fiber optics

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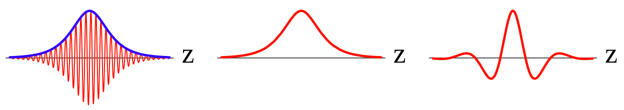
Description

In this project we investigate the propagation of optical pulses in nonlinear dispersive media. The partial differential equations used so far are generalizations of the classical nonlinear Schrödinger equation. We aim on including linear nonlocal operators to cover dispersion relations as well as higher order nonlinearities of nonlocal type. Proposed investigations are in the center focus of modern optical technologies, like in data communication or in modern material research.

Optical pulses: "long" versus "short"



- Pulse duration $\gg$ fs
- $E = A(z,t)\exp i(k_0 z - \omega_0 t)$
- Internal dynamics
- Approximate solution

- Pulse duration $\sim$ fs
- Solve directly for $E(z,t)$
- Ansatz $E = E(t - z/v)$
- Formally exact solution

Existence of localized structures (LS)

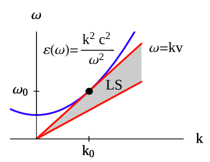
Maxwell equations with a suitable model for $\mathbf{D}(\mathbf{E})$, e.g.,

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\partial_t \mathbf{B}, \quad (E(z,t),0,0) = \mathbf{E}(\mathbf{r},t), \quad D(E) = D_{\text{lin}} + \chi E^3,$$

$$\operatorname{rot}\mathbf{B} = \frac{1}{c}\partial_t \mathbf{D}, \quad \partial_t^2 D - c^2 \partial_z^2 E = 0, \quad (D_{\text{lin}})_\omega = \varepsilon(\omega) E_\omega,$$

with an “arbitrary” $\varepsilon(\omega)$. Ansatz $E = E(\tau) = E(t - z/v)$ yields

$$\hat{L}(\omega) E_\omega \equiv \left[1 - \frac{v^2 \varepsilon(\omega)}{c^2}\right] E_\omega = \frac{v^2 \chi}{c^2} \sum_{\omega = \omega_1 + \omega_2 - \omega_3} E_{\omega_1} E_{\omega_2} E^*_{\omega_3}.$$



- No LS if $L(\omega) = 0$ for some $\omega = \omega_c$
- No LS if $v_{\text{ph}}(\omega) = v$ for some $\omega = \omega_c$
- LS cone is bounded by $v_{\text{ph}} = v_{\text{gr}}$
- $\cosh^{-1}$ solitons for $v \nearrow v_0 = \omega_0 / k_0$.

Universality: $\varepsilon = \sqrt{1 - v^2/v_0^2} \ll 1, \quad E(\tau) = \frac{1}{2} A(\varepsilon\tau) e^{-i\omega_0 \tau} + \text{c.c.}$

$$\varepsilon^2 \left[A(\varepsilon\tau) + \frac{v^2 \varepsilon''(\omega_0)}{2c^2} \frac{d^2 A}{d(\varepsilon\tau)^2} \right] = \frac{3v^2\chi}{4c^2} |A(\varepsilon\tau)|^2 A(\varepsilon\tau).$$

Cooperation

Projects inside Matheon:

D8: Nonlinear dynamical effects in integrated optoelectronic structures (L. Recke, M. Wolfrum)

D9: Design of nano-photonic devices (F. Schmidt, P. Deuflhard)

D20: Pulse shaping in photonic crystal-fibers (A. Demircan)

Applications outside Matheon:

– FhG Institut für Nachrichtentechnik HHI Berlin

– Max Born Institut für Nichtlineare Optik und Ultrakurzzeit-spektroskopie

Model equations for the short pulses

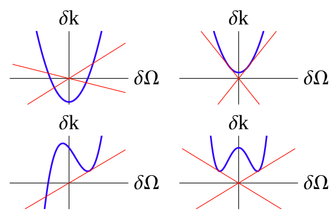
$\delta\omega$	pulse spectral width
$\Delta\omega$	transparency window width
$\bar{\omega}$	mean frequency of the window
$\omega(k)$	dispersion relation

$\delta\omega \sim \Delta\omega$, $\delta\omega \sim \bar{\omega}$: quadratic approximation fails for an extremely short pulse

$$\delta\omega \ll \Delta\omega \ll \bar{\omega}$$

$$\delta k = \frac{\delta\omega}{v_{\text{gr}}} - \frac{\omega''}{2 v_{\text{gr}}^3} (\delta\omega)^2$$

$$\delta k = \frac{\delta\omega}{v_{\text{gr}}} - F(\delta\omega)$$



- Quadratic dispersion: $e^{\pm i\lambda z}$
- Cubic dispersion: no LS
  - Cancellation is possible
  - “Landau damping”
- Quartic dispersion: (?) LS

Examples of the reduced equations:

Classical NSE	$i\partial_z \psi + \alpha \partial_t^2 \psi + \beta |\psi|^2 \psi = 0$
High-order dispersion[a]	$i\partial_z \psi + \alpha(\partial_t^2 \pm \nu^2)^2 \psi + \beta |\psi|^2 \psi = 0$
Integral operator[b]	$P \to \left(1 + \frac{i}{\omega_0}\partial_t\right)^{-1} P$
Partially corrected[c]	$P \to \left(1 - \frac{i}{\omega_0}\partial_t\right) P$
Unidirectional SPE[d]	$\partial_z \partial_t u + u + \partial_t^2 (u^3) = 0$

[a] Solitons with the oscillating tails are possible. [b] A non-local integral operator. [c] Better approximation for the extremely short pulses. [d] An integrable system for the case of special dispersion.

Goals

1. Investigate pulses in a medium, where both non-locality and vector character of Maxwell equations are important.
2. Obtain non-local reduced equations for the optical field and criteria for the existence/stability of the optical solitons.
3. Investigate trains of bounded solitons in a suitable media.

References

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- A. Demircan, U. Bandelow. *“Supercontinuum generation by the modulation instability“*. Opt. Comm. **244**, 185 (2005).
- V. E. Zakharov, E. A. Kuznetsov. JETP **113**, 1892 (1998).