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Optimizing the economic dispatch of weakly-connected mini-grids under uncertainty using joint chance constraints

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Abstract

In this paper, we deal with a renewable-powered mini-grid, connected to an unreliable main grid, in a Joint Chance Constrained (JCC) programming setting. In several rural areas in Africa with low energy access rates, grid-connected mini-grid system operators contend with four different types of uncertainties: forecasting errors of solar power and load; frequency and outages duration from the main-grid. These uncertainties pose new challenges to the classical power system's operation tasks. Three alternatives to the JCC problem are presented. In particular, we present an Individual Chance Constraint (ICC), Expected-Value Model (EVM) and a so called regular model that ignores outages and forecasting uncertainties. The JCC model has the capability to guarantee a high probability of meeting the local demand throughout an outage event by keeping appropriate reserves for Diesel generation and battery discharge. In contrast, the easier to handle ICC model guarantees such probability only individually for different time steps, resulting in a much less robust dispatch. The even simpler EVM focuses solely on average values of random variables. We illustrate the four models through a comparison of outcomes attained from a real mini-grid in Lake Victoria, Tanzania. The results show the dispatch modifications for battery and Diesel reserve planning, with the JCC model providing the most robust results, albeit with a small increase in costs.

1 Introduction

The United Nations' Sustainable Development Goal 7 (SDG-7) urgently calls for universal access to clean energy, a mission that faces challenges despite increased electrification efforts. Projections by the international energy agency estimate that by 2030, around 650 million people might still lack access, and 9 out of 10 will be in West, Central, and East Africa [13]. Electrification via centralized grids is slow and expensive, prompting interest in decentralized mini-grid energy systems that offer quicker deployment and enhanced cost competitiveness [1].

Renewable-powered mini-grids (MGs) as defined in [10, 14] are hybrid electricity supply systems combining wind turbine or photovoltaic (PV) generation (from 10 kW to 10 MW), energy storage systems, and (usually) a Diesel generator into low and medium voltage distribution networks. Classified by their relationship with the main power grid, MGs come in two forms: islanded mini-grids which operate independently, detached from the national (main) network and grid-connected mini-grids which link with the main grid and function as both backup systems for distribution as well as standalone units.

This article focuses on the operational intricacies of grid-connected mini-grids, situated within the urban and peri-urban areas of Sub-Saharan African countries lacking in energy access. The paradox lies in the fact that, despite the presence of central-grid infrastructure, an alarming number of households, numbering in the hundreds of millions, still grapple with the challenge of accessing less than four hours of electricity per day [25]. To counter this issue, governments in these countries are

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progressively adopting grid-connected mini-grids in areas with unreliable main grid supply. Yet, technical uncertainties inherent to grid-connected mini-grids in this context stem from multiple sources: stochastic solar power and demand forecast errors; absolute uncertain main grid outage onset times; and outage durations subjected to statistical analysis. As operational decisions are taken prior to the observation of the uncertainty, it becomes imperative to adopt suitable modeling methodologies that can effectively incorporate all these diverse forms of uncertainty.

Thus, our primary research aim focuses on addressing the four mentioned uncertainties through modeling and algorithmic approaches, utilizing Joint Chance Constraints. Additionally, the study aims to assess the effectiveness of the proposed scheduling strategy by applying it to a case study example of a mini-grid in Lake Victoria.

Introduced by Charnes and Cooper [5], chance constraints offer an appealing tool for dealing with uncertainty in the constraints of an optimization problem. A classical and fundamental introduction to the theory and numerical treatment of chance constraints can be found in Prékopa [22]. A modern treatment is provided in [26]. Since their introduction, chance constraints have become common for economic dispatch problems, notably in hydro reservoir management (see, e.g., [23, 3, 19, 29]) but also in power dispatch (see, e.g., [12, 21]). For mini-grid or micro-grid dispatch, the approach has predominantly involved the use of purely deterministic predictive models [10, 20]. There is increasing interest, however, in applying probabilistic models in order to ensure sufficiently safe satisfaction of demand in a highly stochastic environment, not only with respect to renewable energy but also with respect to instabilities of the main grid. In [30] and [18], the interesting concepts of *probability of self-sufficiency* and *probability of successful islanding*, respectively have been introduced, in order to model the self-sufficiency of a mini-grid when isolated from the main grid by an outage. These concepts rely on keeping reserves for Diesel employment and battery discharge such that an outage of the main grid, to which the mini-grid is connected, can be survived based on these reserves with sufficiently high probability. The shortcoming of the models in [30, 18] is that they use individual (separate for each time in the given interval) chance constraints. While such a model is comfortable to handle since the chance constraint can be transformed without effort into an explicit equivalent, it does not really reflect the wish for robust self-sufficiency. Indeed, even if one may guarantee that self-sufficiency holds true at each time individually with high probability, the probability of violating self-sufficiency at **some** time may be high as well (or: the probability of guaranteeing self-sufficiency throughout a given period of time may be small). This is why we will rather consider **joint** chance constraints in this paper which are, however, more difficult to deal with. Indeed, they require the consideration of probabilities and their sensitivities with respect to the decision vector under multivariate random distributions. For numerical solutions related with joint chance constraints, we shall make use of the so-called *spherical-radial decomposition* of Gaussian random vectors which efficiently applies to Gaussian, Gaussian-like (e.g. multivariate log-normal, Gaussian mixture) or elliptically symmetric (e.g. multivariate Student) distributions and has found a lot of applications both in operations research and optimal control under PDEs with random coefficients, see, e.g., [7, 11, 8, 3]). The possibly striking difference between individual and joint chance constraints has been highlighted, for instance, in [28, p. 547] and [3, p. 34].

In this paper we focus on the challenges arising from the model with joint chance constraints. Therefore, we keep some other modeling aspects simple. In particular, we will not include binary decisions (simplified model for Diesel generator), we will not adequately model realistic battery ageing by means of differential equations and we will keep all decisions to be static with respect to the unfolding of uncertainty over time (thus ignoring the gain of information based on random observations prior to decision taking). The inclusion of all these aspects is subject of current and future work (e.g., [9]).

This article makes two key contributions, namely to the realm of applied joint chance constraint programming and to the advancement of SDG-7. More precisely, our analysis involves the following steps:

- We base our input data on real power measurement data obtained from an operating MG in Tanzania and use forecasts for the solar PV generation and the electricity demand of the connected households.
- We propose a model for an economic scheduling strategy of an MG that is connected to a weak main grid, while accounting for uncertainties of 1) forecasting the renewable power generation, 2) forecasting the demand profiles, as well as uncertainties of 3) the frequency and 4) duration of an outage with a given (high) probability level using joint chance constraints.
- We compare the proposed JCC model with three alternatives. We contrast results with an Individual Chance Constraint, Expected Value Model and a model ignoring outages and uncertainties.

2 Mini-grid topology and modelling assumptions

Fig. 1 illustrates the grid-connected mini-grid configuration. This arrangement includes a solar PV generation unit, one battery energy-storage system (BESS), one Diesel genset, and one connection to an unreliable main-grid.

The PV and BESS are integrated into the system via two distinct inverters linked to the Alternating Current (AC) bus. Interactions with the main grid occurs at the Point of Common Coupling (PCC). At the PCC, a transformer modifies the voltages, translating them from the lower level of the mini-grid to the higher-level of the primary grid (referred to hereafter as main grid or higher-level grid). In the examined context of this paper, this connection to the main grid is unreliable, with outages occurring frequently due to failures in the main grid. This could be an overloading of the main grid, an uncurtailed excess generation or technical aspects due to failures of control strategies, physical components or transmission lines overheating.

Lastly, the Diesel genset, acting as a supplementary component to bridge any electricity supply gaps, generates AC current directly, injecting it into the AC bus.

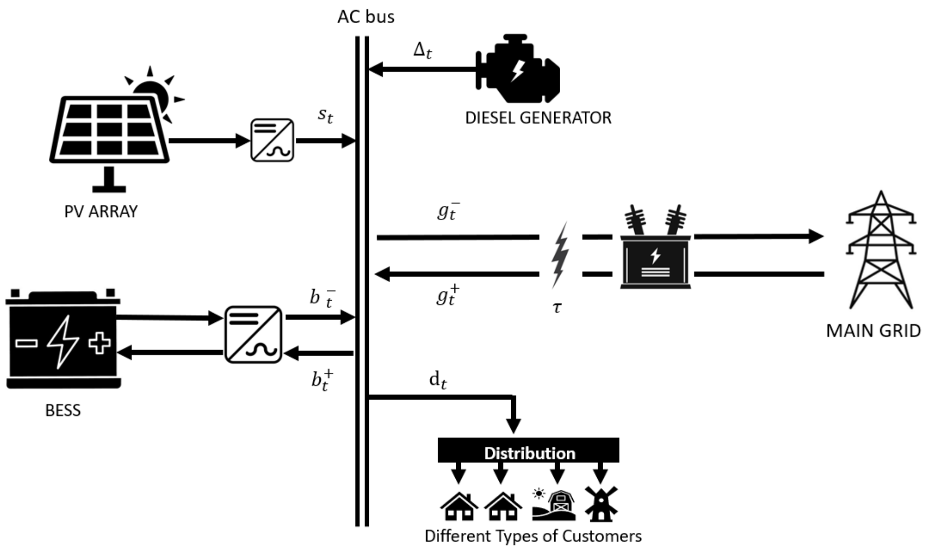

Figure 1: Weakly Grid-connected Mini-grid Topology

Since the MG is balanced through an AC bus, both active and reactive powers will flow to the loads. In this paper, for simplicity purposes, we assume that the loads of all connected households maintain a purely resistive nature. We, therefore, conduct all our modelling using only active power flow figures, neglecting all reactive components.

Further on, we neglect any temperature effects on the MG components and operation, especially for the battery storage system. The latter is modelled as one large synchronized unit that can either be charged or discharged, thus neglecting specific dynamics between the battery packs. Additionally, we model charging and discharging losses in the battery inverter as well as the battery's internal resistance as single, average efficiency parameters.

In terms of the flexibility of dispatch, we assume that the Diesel genset is able to run between 0 and a maximal capacity, without any ramping constraints or restrictions on minimal resting and running times. Moreover, we follow the premise that the MG operator always maintains a continuous fuel supply.

Additionally, we assume that the solar PV output can be controlled with the help of a Charge Controller (CC). In case of an excess of PV generation and an outage preventing any export to the main grid, the PV output can be curtailed fast enough such that the overall local power balance can be withheld.

To model the demand-side, we aggregate all electricity loads into one node, thus neglecting any line losses that occur at the local distribution level.

Finally, we assume that the MG operator has access to time series forecasting of solar power generation and aggregate demand for the forthcoming 24 or 48-hour time interval. Furthermore, we assume that, when an outage of the main grid occurs, the power exchange at the PCC drops to 0, thus preventing any planned imports or exports. Outages can occur with a given probability at any arbitrary moment, the precise timing of which remains uncertain. We further assume that there is statistical evidence about the average duration of an outage from the main-grid [15].

Table 1 summarizes the uncertainties present at the examined MG and assumptions made about their modeling.

Table 1: Mini-grid uncertainties

Source of uncertainty	**Assumptions**
Aggregate demand	access to a forecasting time series for the forthcoming 48 hours
Solar PV generation	access to a forecasting time series for the forthcoming 48 hours
Timing of main grid outages	unknown
Duration of main grid outages	access to statistical evidence as an exogenous parameter [15]

3 Mathematical models

This section defines the mathematical model used for the above described MG topology. As mentioned before, our aim is to find the optimal dispatch of a weakly-connected MG which allows one to survive a failure of the main grid by means of local energy supply (battery, Diesel). In order to achieve this goal, in addition to the planning of a regular dispatch for strongly connected MGs, reserves for Diesel and battery are included into the model. These can then be employed in the islanded mode of the MG in order to meet the local power demand. Since dispatch has to be planned prior to observing randomness in demand, solar radiation and outages, methods of stochastic optimization come into play. Inspired by the work in [30, 18], we feel the application of probabilistic or chance constraints to be most appropriate here. They provide the possibility to find an optimal dispatch including reserves for Diesel and batteries such that the local demand in an islanded mode caused by an outage is met with a given probability (typically close to one). However, as mentioned in the introduction, these papers handle probability individually in time. While this approach eases drastically the computational treatment of chance constraints, it does not guarantee robust demand satisfaction **throughout** the critical time interval. Therefore, our main contribution is devoted to model the MG dispatch problem with respect to joint chance constraints. This type of constraints cannot directly be converted into explicit linear constraints as in the individual case, but requires to deal with probability functions and their derivatives under multivariate distributions.

This section is organized as follows: after introducing the used nomenclature, we present the objective function and the (common) deterministic constraints for the optimization problem. Then, we introduce our additional joint chance constraints with respect to decisions on keeping reserves. We shall assume that an outage may occur at most once a day with a given probability. Its initial time is supposed to be uniformly distributed over the day. As for the duration of the outage, we either fix it (e.g., as a statistical mean from available historical data) or consider it to be an exterior parameter which can be varied in different computations. Next, we move on to a model with individual chance constraints as used in [30, 18], in order to illustrate the gain in robustness by using joint chance constraints. Finally, for the sake of completeness, we also mention two further simplifications of the model: first, the expected-value model in which we replace all random parameters by their expected values, such that statistical information is reduced to the first moment. Second, the regular model, which in addition to the previous simplification, also completely ignores the possibility of outages.

3.1 Objective functions

In this subsection, we will introduce the objective function used across the mathematical models starting with the cases of a strongly-connected and a weakly-connected mini-grid.

Strongly-connected mini-grid

We first present the objective function for a strongly-connected mini-grid, i.e., where outages do not occur. In such a mini-grid, if the operator faces forecasting errors in solar and load predictions, the impact is negligible from a physical standpoint. The system can be adapted by managing exports and imports. However, economically, it's important to note that instantaneous grid exchanges come with a higher cost compared to day-ahead exchanges.

For such a problem, we want to maximize the profit of the MG dispatch, which is equivalent to minimizing the net operational costs minus net revenue. The summation term in the objective (1) reflects the negative profit of the MG over the nominal time horizon T, plus the duration of an outage κ, as the outage could start at the end of the nominal time horizon.

$$f_0(\Delta, b^+, b^-, g^+, g^-) := \tag{1}$$
$$\sum_{t=1}^{T+\kappa} c_{\Delta,t} \cdot \Delta_t + c_{b,t} \cdot (b_t^+ + b_t^-) - p_{d,t} \cdot d_t + c_{g,t} g_t^+ - p_{g,t} g_t^- + \mathbb{E}[c_{\gamma,t}(\gamma_t(\xi_t, \delta_t))].$$

The aim is to maximize the profit of the MG dispatch, i.e., minimize the function f_0, where revenues from electricity sales are subtracted from the marginal costs of operation. The marginal costs consist of the fuel costs $c_{\Delta,t}$ for the Diesel genset, of the battery lifetime depreciation costs $c_{b,t}$ due to the discharge-charge cycling, and of the costs $c_{g,t}$ for importing from the main grid. Solar power is assumed to have zero marginal costs and therefore doesn't appear in the objective function. Thus far, maintenance costs are not explicitly considered. They could be levelized and included in the cost coefficients of each technology. However, for this paper, further considerations about the maintenance approach are omitted. The revenues are generated from selling the electricity at exogenous price of $p_{d,t}$ to the connected households consuming electricity at the MG level, as well as from exporting excess electricity at the exogenous price of $p_{g,t}$ to the main grid. The random variable

$$\gamma_t(\xi_t, \delta_t) := d_t + \delta_t - s_t - \xi_t - \Delta_t - b_t^- + b_t^+ - g_t^+ + g_t^- \quad \forall t = 1, \dots, T + \kappa. \tag{2}$$

is defined as the compensating action establishing the load balance after applying the decisions on the employment Δ_t, b_t, g_t of Diesel, battery and grid exchange on the day ahead market and observing the random variables solar energy $s_t + \xi_t$ and demand $d_t + \delta_t$ (forecast plus random deviation in both cases). A positive value of γ_t represents a lack of energy to be compensated by instantaneous import from the main grid (at a price that may be substantially higher than for the import on the day ahead market). A negative value of γ_t refers to an excess of energy which has to be instantaneously exported to the grid, again at certain costs (so that, contrary to the day ahead market there is no reward for the export). As γ_t and, hence, its associated costs, are random, we minimize the expectation of these costs.

All costs and prices are defined as time-variable parameters, hence the index t for each of these parameters.

Weakly-connected mini-grid

Here, we introduce the intricacies associated with a weakly-connected MG, i.e. with potential outages in the main grid and adjust the objective function accordingly. Starting with the initial objective function (1), we adapt the objective function f_0 to account for the case of possible grid outages. We introduce the following definition:

$$\begin{gathered} f_1(\Delta, b^+, b^-, g^+, g^-, r^\Delta, r^b) := \\ \sum_{t=1}^{T+\kappa} c_{\Delta,t} \cdot \Delta_t + c_{b,t} \cdot (b_t^+ + b_t^-) - p_{d,t} \cdot d_t \\ + \frac{\omega}{T} \sum_{\tau=1}^{T} \left(\sum_{t=1, t\neq\tau,\dots\tau+\kappa}^{T+\kappa} c_{g,t} g_t^+ - p_{g,t} g_t^- + \mathbb{E}[c_{\gamma,t}(\gamma_t(\xi_t, \delta_t))] + \sum_{t=\tau}^{\tau+\kappa} c_{\Delta,t} r_t^\Delta + c_{b,t} r_t^{b-} \right) \\ + (1-\omega) \sum_{t=1}^{T+\kappa} \left(c_{g,t} g_t^+ - p_{g,t} g_t^- \right) + \mathbb{E}[c_{\gamma,t}(\gamma_t(\xi_t, \delta_t))]. \end{gathered} \tag{3}$$

The first term is defined in analogy to (1) including again a summation over the Diesel and battery cycling costs minus the revenues generated by selling the electricity to local customers at a given price. Now, considering the possibility of grid outages, we delineate two scenarios for the costs and revenues associated with grid exchange: one when an outage occurs and another when it does not. These scenarios are articulated in the second and third terms of (3). We make the assumption that, at most, one outage may occur during the designated day ahead, and the probability of its occurrence is denoted as ω. In the absence of an outage during the specified day (probability equals $1 - \omega$), the sole additional costs stem from trading with the main grid (third term). Conversely, when an outage

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occurs, suppose at time τ, then in the interval $[\tau, \ldots, \tau + \kappa]$, representing the duration of the outage, no trading costs with the main grid are incurred (due to the lost connection). However, additional costs arise from utilizing reserve capacities r_t^{Δ}, r_t^{b-} for Diesel generation and battery discharge to meet the local demand in the islanded mode (second term).

We assume that τ is uniformly distributed, indicating that an outage may initiate at any time with equal probability. Consequently, in the second term of (3), we compute the average costs across all potential starting times for the outage. In contrast, the duration κ of the outage is regarded as the statistical mean derived from historical data or treated as an exogenous parameter of the problem, capable of assuming multiple values.

3.2 Deterministic constraints

The physical constraints represent well-known formulations for mini-grid economic dispatch models, consisting in capacity constraints of the MG components, a power balancing constraint for MG supply and demand, inter-temporal constraints for the storage state of charge, as well as considerations of conversion and distribution losses expressed through average efficiency parameters. These will be discussed in more detail in the following.

Capacity constraints of the Diesel genset: it is assumed that the Diesel genset can operate between 0 and its maximal capacity Δ^{max}. Therefore, the sum of the decision variables for the Diesel genset dispatch Δ_t and the Diesel genset reserve r_t^{Δ} have to be within these bounds at all time (4).

$$0 \leq \Delta_t + r_t^{\Delta} \leq \Delta^{max}, \qquad \forall t = 1, \ldots, T + \kappa. \tag{4}$$

Capacity constraints of the battery charge and discharge: similarly to the Diesel genset, the battery charge and discharge rates are also bound by a lower level of 0 and an upper level b^{max}. In the case of discharging the battery, we define the discharge rate as the sum of the actual dispatch decision b_t^- and the planned reserve r_t^b (5). Since we assume that the reserves are used to compensate an electricity shortage and that any excess generation from solar PV can be curtailed fast enough, we omit the definition of a reserve for the battery charging, thus obtaining the constraint (6) for the decision variable of battery charging b_t^+.

$$0 \leq b_t^- + r_t^b \leq b^{max}, \qquad \forall t = 1, \ldots, T + \kappa. \tag{5}$$
$$0 \leq b_t^+ \leq b^{max}, \qquad \forall t = 1, \ldots, T + \kappa. \tag{6}$$

Capacity constraint for the grid exchange: the connection to the main grid at the PCC is also constrained by a certain capacity limit over time. Thus, the decision variables for the grid export g_t^- and the grid import g_t^+ are also bound by an upper value of g_t^{max}:

$$0 \leq g_t^+ \leq g_t^{max}, \qquad \forall t = 1, \ldots, T + \kappa. \tag{7}$$
$$0 \leq g_t^- \leq g_t^{max}, \qquad \forall t = 1, \ldots, T + \kappa. \tag{8}$$

Power balancing constraint: as in any power system, to maintain the system frequency and prevent any voltage surge or sag that would lead to a grid failure, the supply and demand of electricity must be equal at any time. As mentioned in Section 3.1, this balance is established by means of the compensating action γ_t defined in (2), where any current excess or deficit in the load balance is rectified by instantaneous (potentially costly) grid exchange. In an attempt to keep this compensating action small, it is often required in addition, that the unit commitment guarantees the load balance to be satisfied with respect to the forecasts s_t, d_t of uncertain solar energy and demand profiles:

$$s_t + \Delta_t + g_t^+ + b_t^- - b_t^+ - g_t^- = d_t, \qquad \forall t = 1, \ldots, T + \kappa. \tag{9}$$

Substituting this additional relation into (2), one obtains the consequence, that the compensating instantaneous trading with the grid is nothing but the difference between forecasting errors for solar energy and demand:

$$\gamma_t = \delta_t - \xi_t, \qquad \forall t = 1, \ldots, T + \kappa. \tag{10}$$

Capacity, cycling and inter-temporal constraints for the battery energy-storage system: in addition to the constraints on the battery discharge and charge rates defined in constraints (5) and (6), we need to defined additional constraints on the battery energy fill level, also called State Of Charge (SOC), and defined in the following constraints as SOC_t. The first constraint concerns the upper and lower bounds of the SOC which are defined by both physical constraints of the actual battery size, as well battery protection measures that set tighter conditions on these bounds (SOC^{min} and SOC^{max} in the constraint (11)) in order to protect the battery from overheating and extend its lifetime.

$$SOC^{min} \leq SOC_t \leq SOC^{max}, \qquad \forall t = 1, \ldots, T + \kappa. \tag{11}$$

In the second constraint for the battery energy-storage system, we set a condition to the cycling of the battery over the nominal time horizon T, which guarantees that battery's SOC at the the end of each time horizon T is equal to the SOC at the beginning of the time horizon as defined in the constraint (12). With this, we make sure that the dispatch is optimized with the ulterior operation of the MG, beyond the nominal time horizon, is considered and the SOC at the end of the time horizon is not exhausted for subsequent operational time steps. This constraint is optional, since, in practice, the optimization of the MG operation is usually conducted in a rolling horizon approach, preventing such exhaustion of the SOC at the end of an optimization horizon.

$$SOC_T = SOC_0. \tag{12}$$

In the next constraint for the battery energy-storage system, we look into the inter-temporality of the battery discharge and charge processes, which translates into an increase or a decrease of the SOC over time. For this, we define the constraint (13), which relates the SOC at time t to the SOC at time $t-1$, with the initial value SOC_0 given. The relation between SOC_t and SOC_{t-1} takes into account the battery discharge b_t^- and charge b_t^+ multiplied by their respective average efficiency coefficients η^+ and η^-. Since b_t^- and b_t^+ are power figures, expressed in kW or W and the SOC is an energy figure expressed in kWh or Wh, the net charge and discharge rate must be multiplied with the nominal time-step Δt.

$$SOC_t = SOC_{t-1} + (\eta^+ b_t^+ - \eta^- b_t^-)\Delta t, \qquad \forall t = 1, \ldots, T + \kappa. \tag{13}$$

Finally, we define the constraint (14) to describe the potential discharge of the battery, and therefore the potential SOC, in case of an employment of the battery reserves because of an outage. This potential SOC is defined as the SOC at time t (SOC_t) minus the cumulative reserves r_s^b that are planned in the κ previous time steps, multiplied by the average discharging efficiency η^- and the nominal time-step Δs. Since r_s^b is a non-negative decision variable, the constraint (14) entails the definition of the lower bound presented in the constraint (11).

For $t = 1, \ldots, T + \kappa$:

$$SOC^{min} \leq SOC_t - \sum_{s=\tau}^{\min\{\tau+\kappa,t\}} \eta^- r_s^b \Delta s, \quad \tau = 1, \ldots, \max\{t - \kappa,\, 1\}. \tag{14}$$

Since we consider a battery reserve only for the discharging, we do not need to define an adjusted constraint for the upper bound of the potential SOC. Therefore, the definition of the upper bound in the constraint (11) remains unchanged.

3.3 Random parameters

In this section, we describe how the random parameters, namely solar energy and demand are dealt with in our model.

Time series analysis

Apart from the fixed data, two data sets are used in this study: solar data in dry season and rural load data. In this study, we implement linear ARIMA modelling [4] to forecast the next 48 hours (such that we can vary κ to explore different scenarios). We used the auto.arima() function implemented in the R software [24] to select the best parameters. We then refined the model by manually increasing the autoregressive (AR) and moving average (MA) components from significant lags in the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots. Model performance was evaluated using Root Mean Square Errors (RMSEs) and coefficient of determination R^2.

Establishing the stationarity of the time series is a fundamental prerequisite for ARIMA. To determine the necessary number of differences to attain stationarity, we employed the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS test).

For the rural load data the best model we obtain (with the lowest Akaike Information Criterion) is an ARIMA$(24, 1, 27)$. The comparison between the actual value and the forecasting value for the 48 points out-of-sample is given in Figure 2.

Similarly, the best model we got for the solar power production during the dry season was an ARIMA$(12, 0, 13)$ and the forecasted values for a 48 hours test set are shown in Figure 3.

We conducted a comprehensive statistical residual analysis on the ARIMA models to evaluate the appropriateness of the model fit. Our investigation revealed that the residuals from rural load, denoted as δ_t, from solar forecasts, denoted as ξ_t, conform to a normal distribution with zero mean. The Shapiro test resulted in a p-value of 3.2×10^{-3} for load and of 6.4×10^{-5} for solar.

These observations hold particular significance as one of the crucial theoretical and numerical aspects concerning our joint chance constraint problem pertains to the convexity of the set of decision variables satisfying (3.4).

It is a well-established result ([22], Theorem 10.2.1) that this set exhibits convexity under the condition that the law of $\delta + \xi$ constitutes a log-concave probability measure on $\mathbb{R}^{T+\kappa}$. This finding aligns with

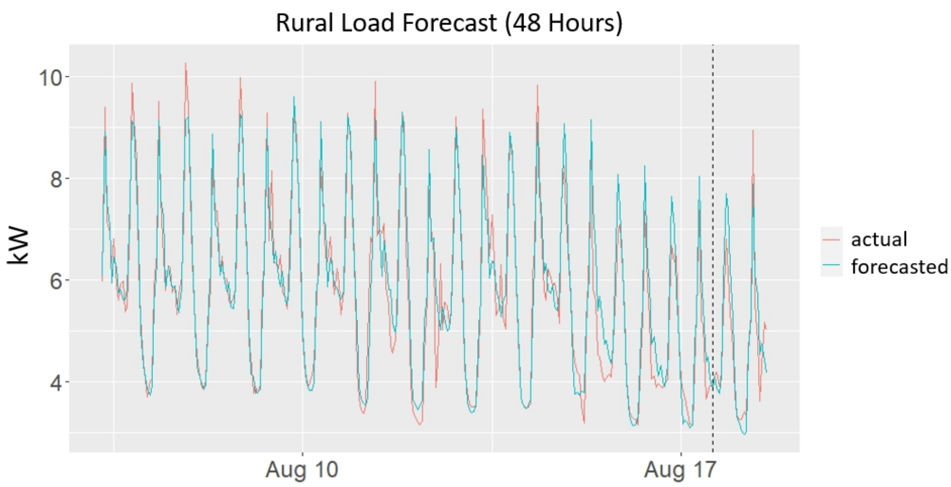

Figure 2: Actual and Forecasted ARIMA$(24, 1, 27)$ Load model

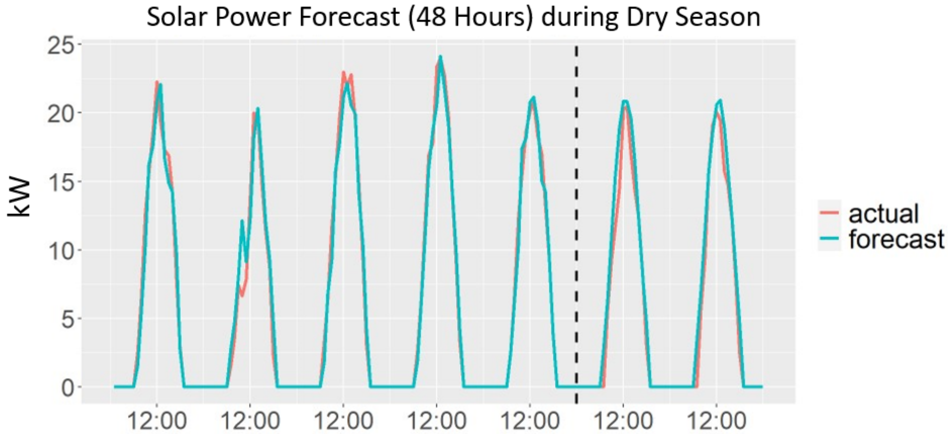

Figure 3: Actual and Forecasted ARIMA$(12, 0, 13)$ Solar Power model

Prékopa's theorem, which asserts that the law of $\delta + \xi$ is log-concave whenever $\delta + \xi$ possesses a log-concave density, as is the case for the multivariate normal distribution. Leveraging the insights from our residual analysis and the combination of these theorems, we are able to confirm the convex nature of our joint chance constraint problem.

Expectation of the costs for instantaneous grid exchange

The above-mentioned fact that both ξ_t and δ_t are centered normally distributed random variables implies that so is γ_t by (2). We shall assume in this paper that the cost function for instantaneous grid exchange has the concrete form

$$c_{\gamma,t}(z) = \begin{cases} c^*_\gamma z & \text{for } z \geq 0 \quad (c^*_\gamma > 0) \\ 0 & \text{else} \end{cases} .$$

This means that costs for importing unsatisfied energy grow linearly with the amount, whereas excess energy can be got rid off at no costs. From here, exploiting that γ_t is normally distributed with mean zero and some standard deviation σ (which follows from the standard deviations of ξ_t and δ_t), it is easily derived that

$$\mathbb{E}[c_{\gamma,t}(\gamma_t(\xi_t,\delta_t))] = c_\gamma^* \frac{\sigma}{\sqrt{2\pi}}.$$

Consequently, this constant term in the objective functions (1) and (3) does not influence the solutions of the according optimization problems but just their objective value.

3.4 Probabilistic and expected-value constraints for considering outages

In this section we present different constraints on reserves for Diesel and battery in the case of weakly-connected mini-grids affected by occasional outages of the main grid. The presentation is top down according to the quality of models. We start with a model for joint chance constraints, which is at the core of this paper.

Joint chance constraints (JCC)

As explained before, in the case of a weakly connected mini-grid we expect an outage of the main grid at any initial time τ with a given duration κ. The aim of the dispatch in such case is to retain reserves for Diesel and battery in addition to their regular employment such that the mini-grid can survive in an islanded mode without exchange with the main grid. Since all operational decisions are taken prior to observing uncertainty, successful islanding cannot be guaranteed deterministically but only with a given (possibly high) probability. Satisfying at some fixed time t the true demand $d_t+\delta_t$ (as opposed to the forecasted demand d_t) under the true solar power $s_t+\xi_t$ with the help of scheduled local dispatches of Diesel and battery and additional employment of reserves retained for Diesel and battery, but without having access to the main grid, amounts to the inequality

$$s_t + \xi_t + \Delta_t + b_t^- - b_t^+ + r_t^b + r_t^\Delta \geq d_t + \delta_t$$

Note that we cannot formulate an equality for the load balance here because of the present randomness of some of the parameters which is revealed only after taking the dispatch decisions. Substituting d_t from (9) leads to the equivalent inequality

$$\delta_t - \xi_t + g_t^+ - g_t^- \leq r_t^b + r_t^\Delta \tag{15}$$

As mentioned above, we want to satisfy with some high probability p this inequality throughout the time window of an outage starting at time τ and ending at time $\tau+\kappa$. Since the initial time is supposed to be unknown, we formulate a corresponding probabilistic constraint for each τ. Altogether, we obtain the following system of constraints:

$$\mathbb{P}(\delta_t - \xi_t + g_t^+ - g_t^- \leq r_t^b + r_t^\Delta \quad \forall t \in \{\tau, ..., \tau+\kappa\}) \geq p, \qquad \forall \tau \in \{1, ..., T\}. \tag{16}$$

Each single of these constraints is called a *joint probabilistic constraint* because it involves a system (not just one) of random constraints inside in order to guarantee successful islanding uniformly for the whole time window of an outage. As a consequence, one deals with probabilities under multivariate (not just univariate) normal distributions. This is a particular challenge because no explicit formulae for probabilities are available in this case. Moreover, apart from simply approximating the probabilities themselves (which could be done by some Monte Carlo or Quasi-Monte Carlo sampling), the efficient numerical solution of the arising optimization problems based on solvers from nonlinear programming requires sensitivities of these probabilities with respect to decision variables. Efficiently implementable gradient formulae in the case of Gaussian or related distributions are presented in the form of spherical integrals, e.g., in [27].

Individual chance constraints (ICC)

To contrast the JCC model with other more common implementations found in the literature, we introduce in the following the Individual-Chance-Constraints (ICC), which drastically reduce the computational effort but can only ensure the p-level reliability for single time steps and not for the whole outage duration κ.

More precisely, we replace the introduced joint chance constraint system by the following individual chance constraint system:

$$\mathbb{P}(\delta_t + \xi_t + g_t^+ - g_t^- \leq r_t^{b^-} + r_t^{\Delta}) \geq p, \qquad \forall t = 1, \dots, T + \kappa, \tag{17}$$

Observe that, in contrast with (3.4), the probability of successful islanding is considered individually for each time step but no longer uniformly over the time window of an outage. As a consequence, the obtained probability function is univariate normal and can be easily inverted using its p-quantile q_p:

$$r_t^{b^-} + r_t^{\Delta} - g_t^+ + g_t^- \geq q_p, \qquad \forall t = 1, \dots, T + \kappa. \tag{18}$$

In this way, the system (3.4) of nonlinear, non-explicit and potentially difficult to handle joint chance constraints is reduced to a simple system of explicit linear constraints in the decision variables which is easy to handle numerically. The drawbacks of this model will be illustrated in the numerical results below.

Expected-value constraints

Next, we further simplify the model by keeping the assumption of a weakly-connected mini-grid but avoiding probabilistic constraints at all and requiring the constraints for successful islanding to hold just on average. We denote this model in the following as *expected-value dispatch model*. In this case, by passing to expectations in (15) and taking into account that the expectations of forecast errors are zero, one ends up at the following system of constraints describing successful islanding:

$$g_t^+ - g_t^- \leq r_t^b + r_t^{\Delta}, \qquad \forall t = 1, \dots, T + \kappa. \tag{19}$$

Observe that (19) represents the same linear inequality system as (18) but with different right-hand side.

Regular dispatch ignoring outages and forecasting errors

The final model we introduce is the most primitive one in that it ignores both forecasting errors and outage occurrence. It shall be denoted in the following as the *regular model* due to the prevalent trend in the literature, where the operation or economic dispatch of mini-grids typically overlooks uncertainties [2, 6, 16, 17, 10]. These models predominantly adopt a deterministic approach, focusing only on the expected value of the forecasting time series, disregarding their random errors and disregarding blackout uncertainty. The first simplification eliminates the last term of the objective function f_0 presented in (3) that accounts for the random instantaneous exchange with the grid. The second simplification concerns the assumption of a strongly connected mini-grid (outages not present). Under these simplifications, we obtain the minimization objective function presented in (20).

$$f_2(\Delta, b^+, b^-, g^+, g^-) = \sum_{t=1}^{T+\kappa} \Big(c_{\Delta,t} \cdot \Delta_t + c_{b,t} \cdot (b_t^+ + b_t^-) - p_{d,t} \cdot d_t + c_{g,t} g_t^+ - p_{g,t} g_t^- \Big). \tag{20}$$

Furthermore, since the regular model doesn't include any reserve considerations, we adjust the box deterministic constraints for the Diesel and battery discharge variables as shown in (21)-(22), and remove the constraint for the potential state of charge presented in (14). Meanwhile, the constraints (7)-(13) remain unchanged.

Capacity constraints of the Diesel genset

$$0 \leq \Delta_t \leq \Delta^{max}, \qquad \forall t = 1, \dots, T + \kappa. \tag{21}$$

Capacity constraints of the battery discharge

$$0 \leq b_t^- \leq b^{max}, \qquad \forall t = 1, \dots, T + \kappa. \tag{22}$$

4 Case study

Our case study is centered around an operational mini-grid in Chifule, an island of Lake Victoria, Tanzania. Currently, the MG remains isolated, lacking any linkage to the main-grid. However, considering the research question at hand and anticipating a future where many MGs in sub-Saharan Africa will be connected to weakly-connected main grids, we assume that the MG, for the purposes of this study, is connected to a main grid characterized by unreliability and frequent outages.

Physical parameters

The physical parameters pertinent to this context are detailed in Table 2, while the price and cost parameters are presented in Table 3. In conjunction with the parameters listed in these tables, it is assumed that the nominal time horizon is 24 hours, and the time step is set at one hour.

All parameters, excluding those pertaining to the main grid connection, are defined based on data from the MG operator in Tanzania. For the parameters related to the grid connection, we use heuristic and model-informed values. For instance, for the maximum available grid capacity g^{max}, we assume a high value meaning that in case of grid availability, the grid can always guarantee the satisfaction of the local electricity demand. For the average duration of an outage κ, we initially set its value to 3 hours, based on empirical values collected from main-grids in similar geographies [15]. The κ parameter will be varied, further on, to examine its effect on the obtained solutions. For the probability of an outage ω, we assume that outages occur daily, i.e., $\omega = 1$ to magnify the effect of outages on the daily planning of the dispatch.

Monetary parameters

The monetary parameters, presented in Table 3 are also based on indications by the MG operator for the marginal costs of the fuel, of the battery aging and the electricity sale price to local customers. For the cost of importing from the grid and the price of exporting to it, we use values derived from expert interviews with weakly-connected MGs in sub-Saharan Africa. Thereby, we assume that prices and costs differ between night and day, reflecting market conditions, i.e., that importing from the grid costs more during the night than during the day, since at night, the main grid is overall more stressed due o the absence of (cheap) renewable-energy generation (e.g., solar), and that selling to the grid during the day is more valuable due to the higher demand.

Table 2: Physical parameters

	Parameter	Unit	Value	Description
Battery system	η^+	%	95	average battery charging efficiency
	η^-	%	95	average battery discharging efficiency
	σ	%	0	battery self-discharge rate
	b^{max}	kW	10	maximal battery charging/discharging rate
	b_{cap}	kWh	100	maximal battery capacity
	SOC^{max}	%	90	maximal battery state of charge
	SOC^{min}	%	20	minimal battery state of charge
	$SOC(t=0)$	%	35	initial battery state of charge
Diesel genset	Δ^{max}	kW	5	rated Diesel genset capacity
Main grid connection	g^{max}	kW	100	maximum available grid capacity
	κ	-	3	average duration of outages
	ω	-	1	probability of an outage occurring during one day
Solar input	S_t^{max}	kW	see Fig. 4	forecasting of maximum solar production
Meta-parameters	p	%	90	probability of successful islanding
	d_t	kW	see Fig. 4	forecasted electricity load

Table 3: Cost & price parameters

Parameter	Unit	Values	Description
$c_{\Delta,t}$	€/kWh	0.35	marginal cost of Diesel
$c_{b,t}$	€/kWh	0.0055	marginal cost of battery
$c_{g,t}$	€/kWh	0.55 (night), 0.15 (day)	marginal cost of grid import
$c_{\gamma,t}$	€/kWh	0.85 (night), 0.45 (day)	cost for instantaneous grid exchange
$p_{g,t}$	€/kWh	0.08 (night), 0.13 (day)	price for grid export
$p_{d,t}$	€/kWh	0.55	price for electricity sales to customers

5 Results

In this section, we present the results of the optimization problem of the purely deterministic, ICC and JCC models and compare them to investigate the effects achieved due to the different formulations and ways of handling the uncertainties. For these results, we use the previously presented input parameters and time series.

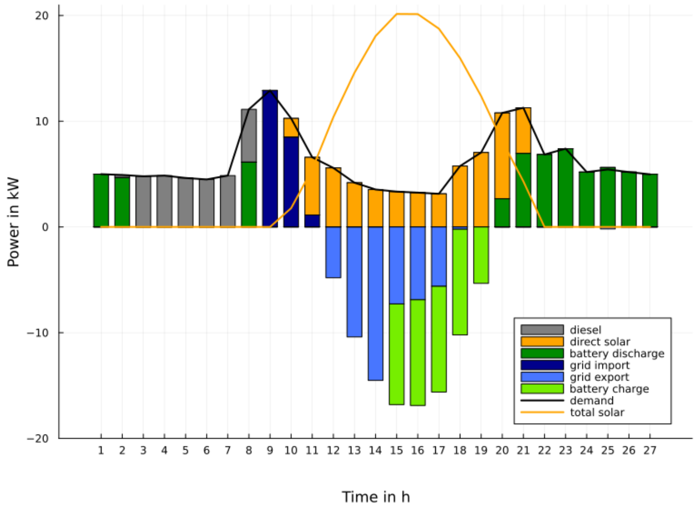

Figure 4: Economic dispatch in the regular model

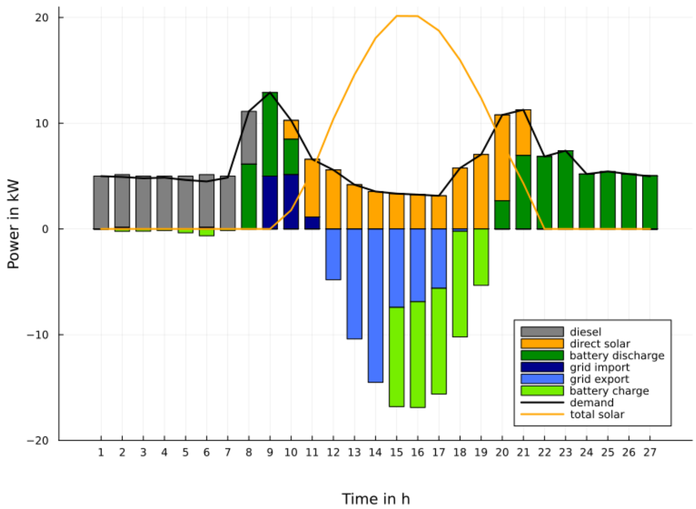

Figure 5: Economic dispatch for the expected-value model

Optimal dispatch

First, we look into the optimal dispatch of each component, in order to meet the fixed electricity demand profile (Figures 4, 5). For the regular model and the expected-value model, the figures show similar patterns of discharging the battery, employing the Diesel genset and to some extent importing from the main grid during the night, while using the solar PV generation during the day with a peak around 3 p.m. The solar power is used to meet the electricity demand directly and to charge the battery or export to the main grid with any excess generation.

The figures also show that the battery is used in the early evening hours after sunset, Diesel in the early morning hours and then around sunrise electricity is imported from the main grid, as it starts getting cheaper (see Table 3). The excess generation from the solar PV system is mostly sold to the main grid for additional revenue generation, while some of it is used to charge the battery in preparation for the night hours. This optimal dispatch leads to expected profits of 84.46 € for the optimization horizon $T + \kappa$, i.e., 27 hours.

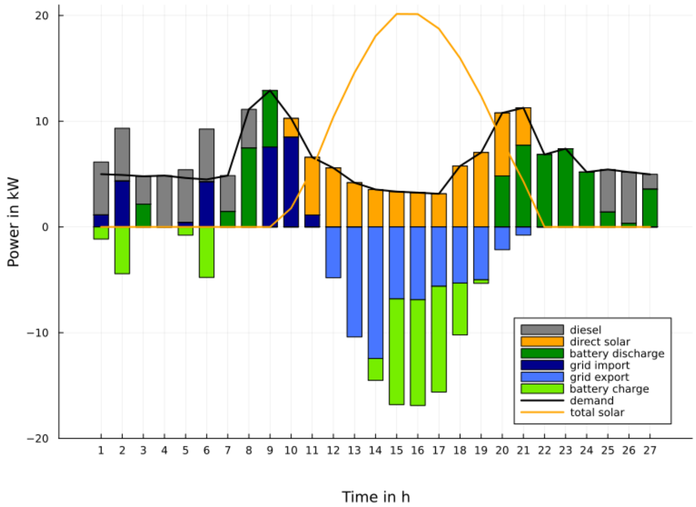

Figure 6: Economic dispatch for the ICC model

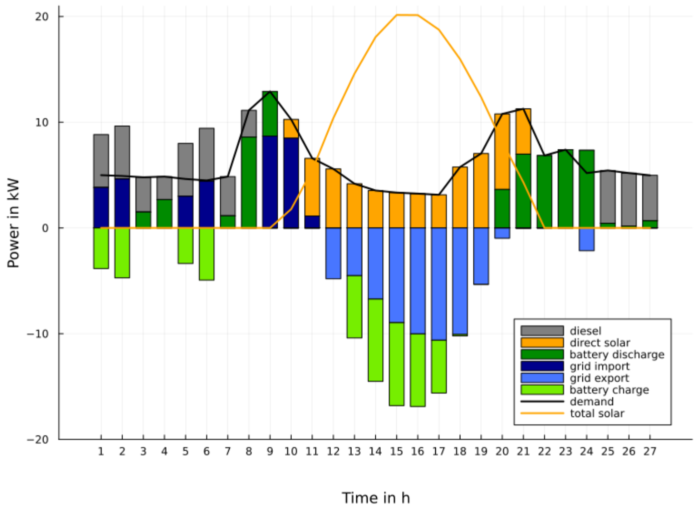

Figure 7: Economic dispatch for the JCC model

Looking into the results of the ICC model, we identify a similar dispatch, with only minor differences from the regular model, mostly occurring during the day in the way that the excess solar PV generation is used (see Fig. 6). In fact, with the individual chance constraints introduced to the model, the optimizer hedges against possible forecasting errors and main grid outages. Thus, the charging of the battery starts sooner than in the dispatch of the model disregarding all uncertainties and, overall, more of the excess electricity is used to charge the battery than to export to the main grid, leading to less electricity quantities sold and thus to less generated revenues. The expected profits drop however by only a few cents, i.e., are about 84.24 €, all while ensuring a high probability of successfully meeting the local demand in an islanded mode and compensating for forecasting errors of 90%.

Still, as explained in earlier sections, the ICC model guarantees the present islanding probability only for the single time step when an outage starts and doesn't cover the entire duration of the outage. For this, we look into the dispatch results of the JCC model, shown in Fig. 7. The figure showcases larger differences than both previous models, mainly in the dispatch from the battery discharging during the early morning hours. During the hours 1-7, more Diesel is deployed, while the battery discharging is

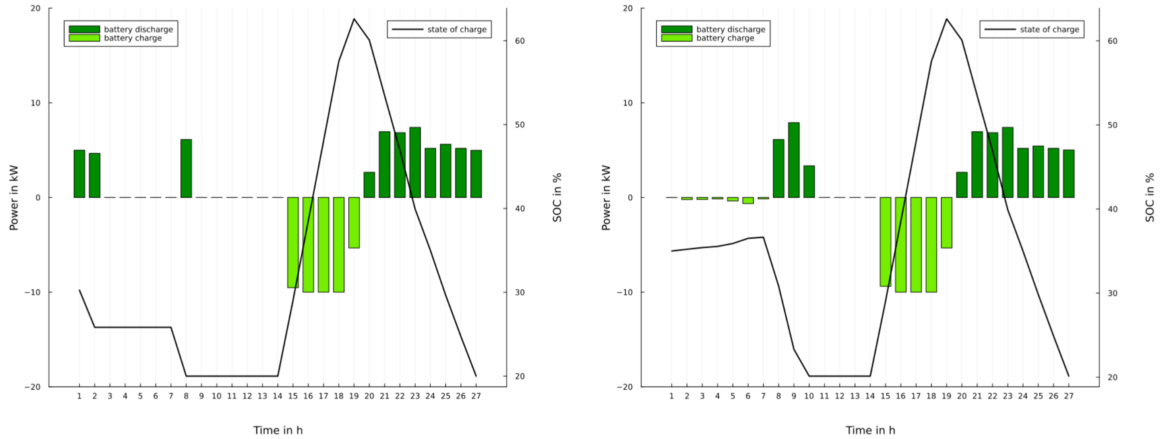

Figure 8: Battery cycle for the regular model (left) and for the expected-value model (right)

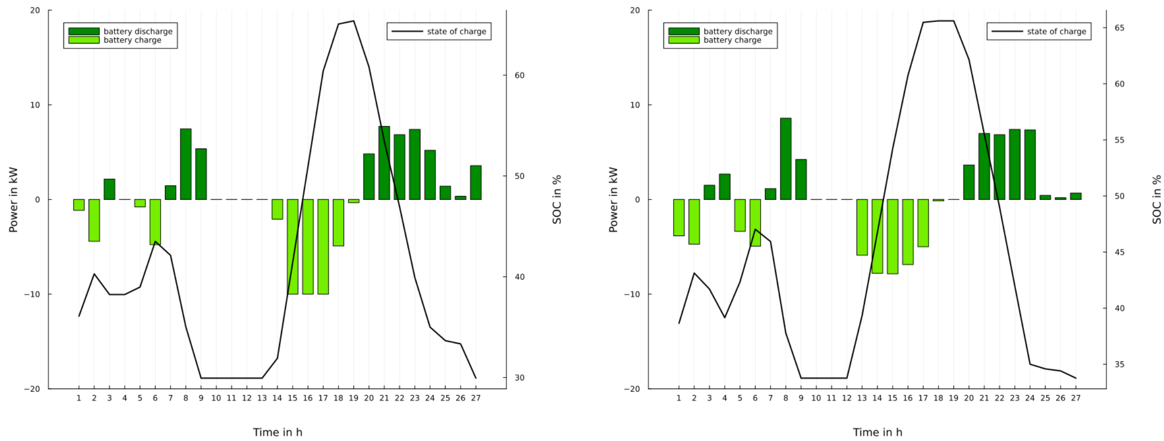

Figure 9: Battery cycle for the ICC model (left) and for the JCC model (right)

significantly reduced and kept as a reserve. Additionally, a lot more excess PV electricity is used during the peak hours to charge the battery, resulting in significantly less quantities exported to the main grid. The expected profit drops to 83.18 € for a probability of $p = 90\%$ and an outage duration of $\kappa = 3$. We present the influence of these two parameters on the profits and dispatch schedule further on in this section.

Optimal battery cycle

The presented differences in dispatch can also be examined in terms of optimal battery cycling strategy, shown in Figures 8, 9 as charging and discharging powers (in different shades of green) and SOC development (as a continuous black line).

While the battery from the regular model, the expected-value model and the ICC model is discharged in the first hours of the day more abruptly, the results from the JCC model showcase a smoother discharging strategy. In addition, larger charging rates are fed into the battery during the peak solar generation in the JCC case than in the ICC case and both are even larger than the purely deterministic case.

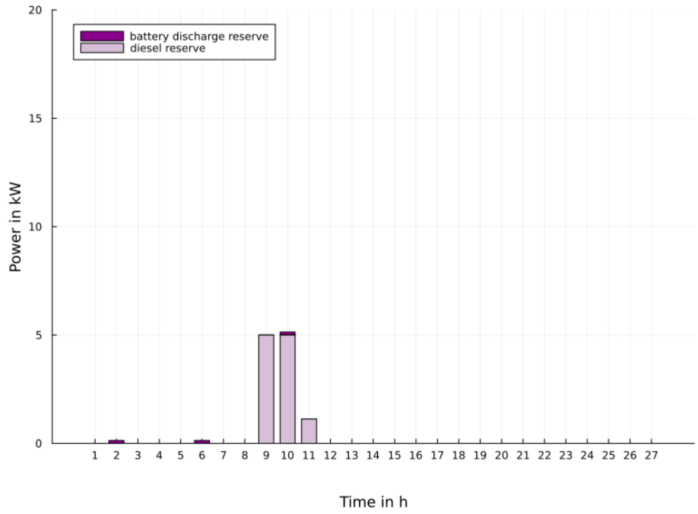

Figure 10: Reserves for the expected-value dispatch model

Optimal reserves

Finally, we take a look at the reserves planned for the entire optimization horizon.

Starting with the reserves in the case of the expected-value model shown in Fig. 10, the reserves match exactly, as expected, the planned net grid exchange at each time step. This is explained by the formulation of the inequality constraints on the minimal reserves. Since the goal is to minimize the cost associated with the deployment of the reserves, the optimizer sets these decision variables to their lower bound values as defined in (19), i.e., the net grid exchange $g_t^+ - g_t^-$.

Fig. 11 presents the planned optimal reserves for the case of an ICC model. Mainly battery discharge capacities are planned as reserves, with the highest levels occurring at times when there is grid exchange in the actual optimal dispatch, for instance, in timesteps 9 and 10. Diesel reserves are more expensive (see Table 3) and are therefore less significant, and are only deployed once the battery discharge rate or state of charge limits are exhausted. During the peak of solar generation in the hours 12-19, where excess electricity is recorded, no reserves are planned.

For the case of a JCC model, Fig. 12 shows that more battery and Diesel reserves are planned in all timesteps and even in more hours of the day than in the ICC case, i.e., only in timesteps 13-18, no reserves are present. These additional possible profits are held back explaining the worsening of the objective function value for the JCC case. However, in the following, we prove the increased reliability of the MG operation that is achieved by the JCC model, justifying the small drop in profitability showcased so far.

Increased reliability

With the ICC and JCC model, we are capable of handling many uncertainties in the MG operation, as explained in Section 2, thus increasing the reliability of meeting the local electricity demand. Thereby, we handle different types of uncertainty: on the one hand, the uncertainties about the forecasting of the solar PV generation and the electricity demand, on the other hand, the uncertainties about an outage of the main grid. The uncertainties about when and how long a grid outage could occur enlarges the uncertainties about the forecasting errors: Provided that the grid is reliable and doesn't experience

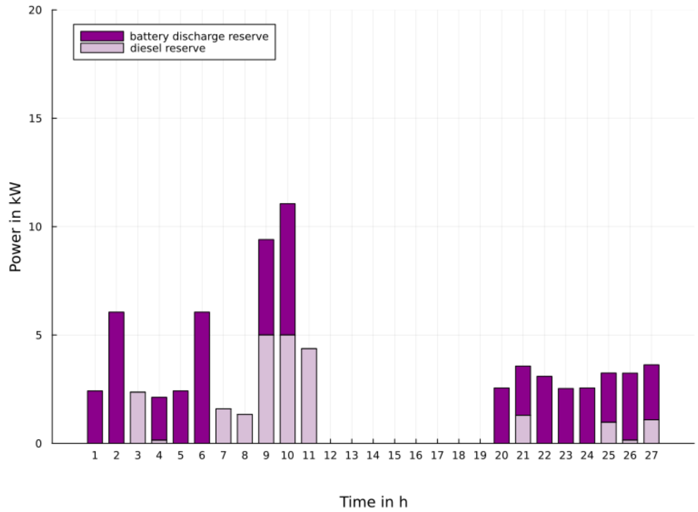

Figure 11: Reserves for the ICC dispatch model

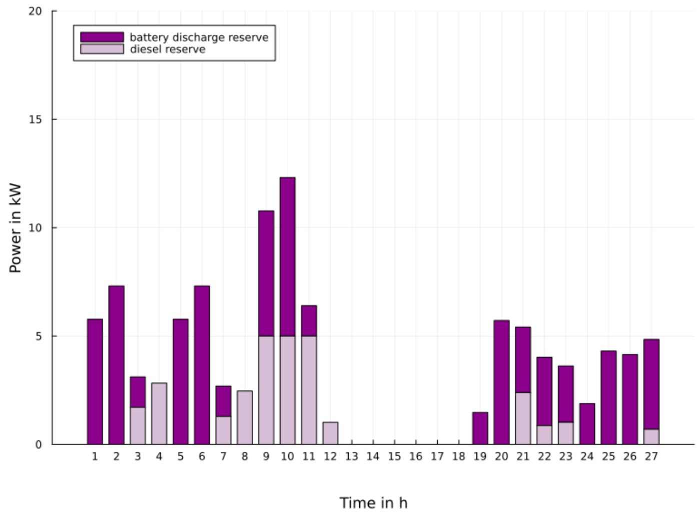

Figure 12: Reserves for the JCC dispatch model

any outages during one day, the main grid could level up any forecasting errors and ensure that the power balance is always met. However, with an unreliable grid, not only are forecasting errors more noticeable, but also any planned grid exchange in the optimal dispatch could be risky and lead to load shedding and an unmet power balance in the MG.

In the previous figures, we prove that at times of a planned exchange with the main grid, enough reserves from the battery and Diesel are kept in the MG and, therefore, the reliability of meeting the local demand in the case of an outage is increased depending on the preset reliability level (compare Fig. 6 and 7 with Fig. 11 and 9, respectively). In the following figures, we show the effect of the forecasting errors and how the additionally planned reserves can also compensate them.

Starting with the ICC case, Fig. 13 showcases, on the left, 10 realizations of the forecasting errors (according to the determined distributions), and on the right, how these errors are compensated using the reserves planned for them. Given that the individual chance constraints are defined as follows and

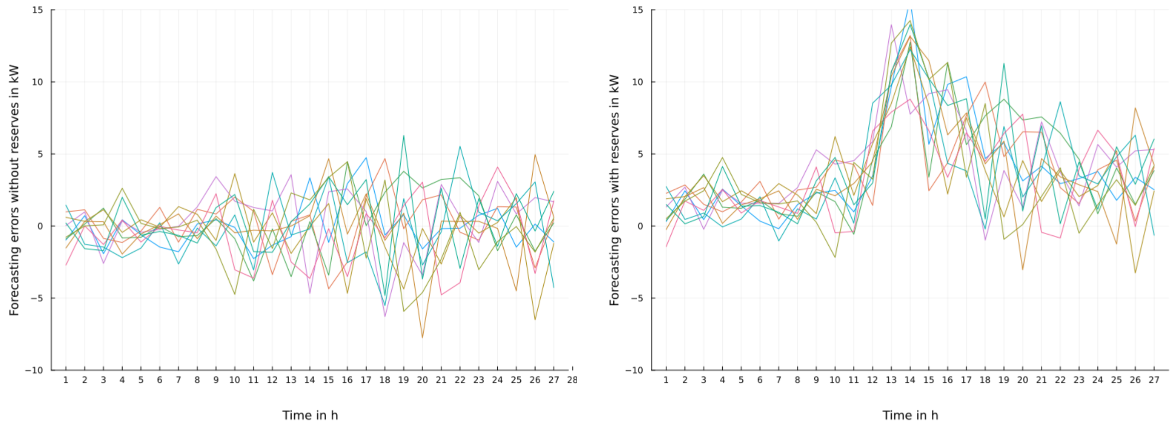

Figure 13: Forecasting errors of solar PV generation and electricity demand without (right) any planned reserves and with reserves (left) in the ICC model

as explained in Section 3.4)

$$\mathbb{P}(r_\tau^{b^-} + r_\tau^\Delta + g_\tau^- - g_\tau^+ - (\xi_\tau + \delta_\tau) \geq 0) \geq p, \qquad \forall \tau \in \{1, ..., 27\},$$

any curves that are below the zero line represent a violation of the probabilistic constraint, and thus unmet demand translating into lost revenues. Hence, with 10 simulations and a probability $p = 90\%$, an empirical approximate of only one curve in each single timestep should be below zero once the chance constraints are applied, which is confirmed in Fig. 13. If we look, however, at windows of outages, we find that several different lines violate the reliability constraint in each time step, hence the probability of reliability over one outage window is less than 90%. This is precisely the reason behind introducing joint chance constraints which will be discussed next.

In the JCC case, we apply the same logic of verifying the reliability of the MG operation, i.e., the capability of the planned reserves to compensate forecasting errors up to a given probability level. We define the chance constraints as follows (also see Section 3):

$$\mathbb{P}(r_t^{b^-} + r_t^\Delta + g_t^- - g_t^+ - (\xi_t + \delta_t) \geq 0 \ \forall t \in \{\tau, \ldots, \tau + \kappa\}) \geq p, \qquad \forall \tau = 1, ..., 24,$$

which means that with joint chance constraints, we look into windows of reliability that range between τ ... $\tau + \kappa$, with $\tau = 1 \ldots T$ to verify that the probability level is withheld throughout the entire window, and that, therefore, only one and the same realization curve is below the zero line for each of the windows. This is proven for all possible starting times of an outage $\tau = 1 \ldots T$, as shown in Fig. 14 with the outage starting times $\tau = 1, 5, 24$, for example.

Table 4 summarizes some key indicators of solving each of the optimization models, i.e., the optimal profit, the type of model and the required solve time. The values are for an optimization model solved with the input parameters presented in Section 4, i.e., a nominal time horizon $T = 24$ and an outage duration $\kappa = 3$ (optimization horizon of 27 steps) and a probability level $p = 90\%$ for ICC and JCC.

On the one hand, the purely deterministic and the ICC models represent linear formulations that are solved by the linear solver GLPK. The ICC includes $T + \kappa = 27$ linear chance constraints. The JCC model, on the other hand, includes $T = 24$ nonlinear chance constraints with multivariate probability functions of dimension $\kappa + 1 = 4$. The JCC optimization problem is solved with the local nonlinear solver IPOPT, using two methods to compute the probability functions and their gradients: Quasi-Monte-Carlo with the Genz algorithm and Spherical Radial Decomposition [27]. Both computation methods yield the same optimal solution with different solve times, presented in Table 4.

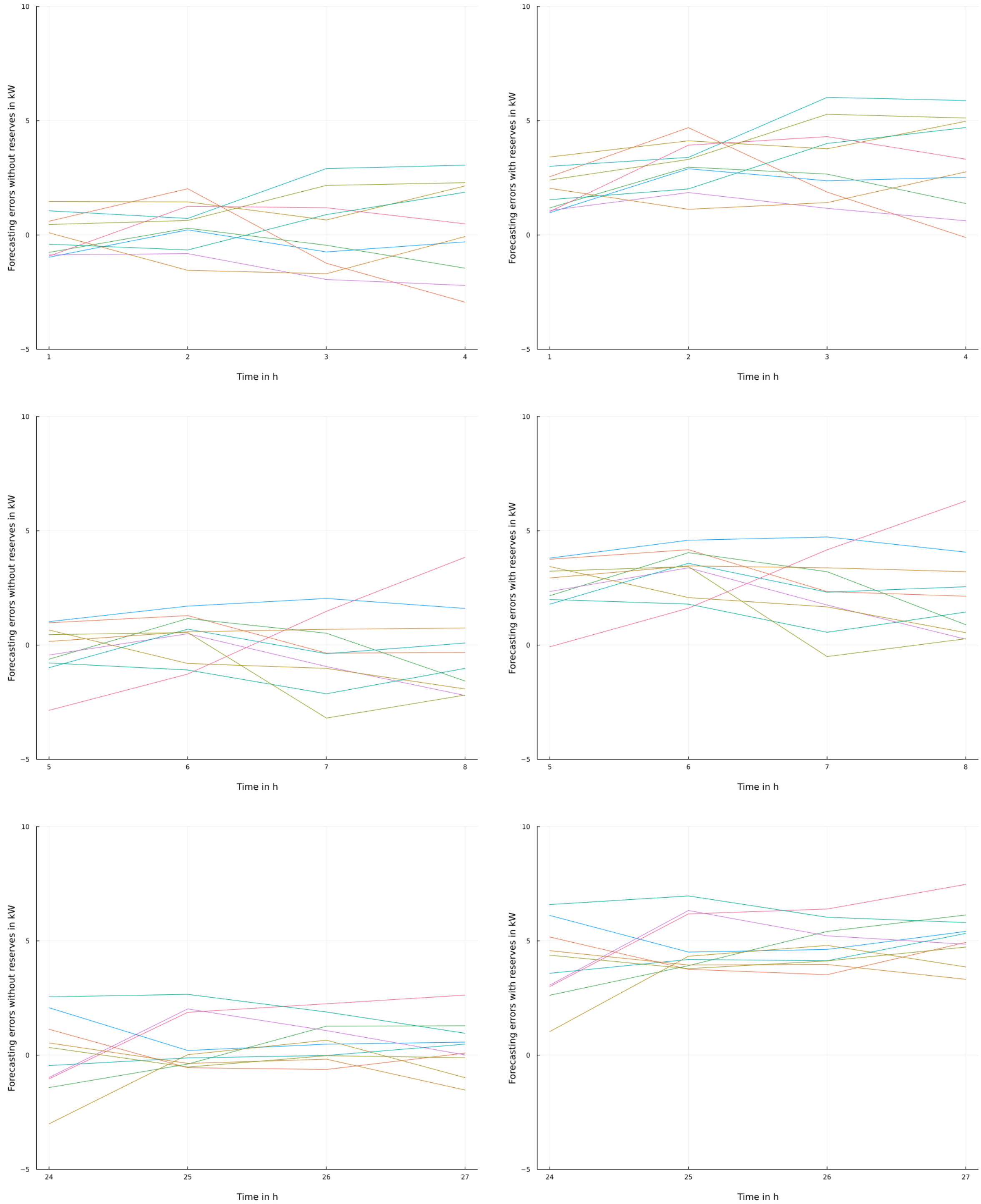

Figure 14: Forecasting errors of solar PV generation and electricity demand (left) and without (right) planned reserves for $\tau = 1$ (top), $\tau = 5$ (middle) and $\tau = 24$ (bottom) in the JCC model with $\kappa = 3$

Table 4: Comparison of the four models

Modelling approach	Profit	Type	Solve time
Regular	84.46 €	Linear	$<$ 1 millisecond
Expected-value	78.00 €	Linear	$<$ 1 millisecond
ICC $(p = 90\%)$	71.84 €	Linear	3-5 milliseconds
JCC $(p = 90\%, \kappa = 3)$	68.73 €	Nonlinear	30-60 seconds [1]

Finally, we investigate the effect of changing probability levels p and changing outage duration κ on the optimal results.

Fig. 15 presents the variation of the optimal expected profits depending on the set probability level. The latter was varied from a value close to 0% up to the maximum probability level above which the problem becomes infeasible. For the present case study, the maximum probability is around 92%. In general, the variation of this parameter, as well as the tendency of the curves depend on the system sizing and the input parameters to the optimization problem.

As shown in the figure, the trend is such that higher p-levels lead to a worse objective function value, since this means an increase of the reliability and, therefore, more hedging against the uncertainty leading to an overall decrease in profits.

Furthermore, compared to the value of the optimal expected profit of the regular model which is around 84.46 €, the ICC and JCC models always lead to less profits, 71.84 € and 68.73 € respectively, since they add additional constraints to the feasible set and therefore lead to a worse objective function value. In addition, due to its tighter constraints and the need for increased planning of reserves in order to survive the whole duration of an outage, the JCC model formulation always has a worse objective function value than the ICC.

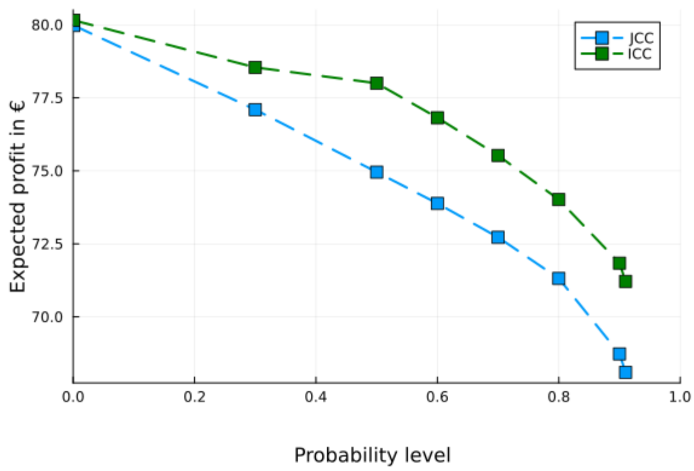

Figure 15: Variation of the objective function for the ICC (green) and the JCC (blue) model formulation depending on the probability level

To investigate the effect of varying κ, we look into the maximal reliability level (referred to hereafter as p-max problem) that is feasible given the sizing of the studied MG as well as the boundary conditions. Formulating the p-max problem requires changing the objective function to a maximization of the probability level p, which is changed from being an input parameter to being a decision variable.

Fig. 16 presents the maximal reliability level for values of the outage duration κ between one and ten. It shows that the higher the outage duration, the lower is the maximal reliability to be achieved. Already with an outage lasting $\kappa = 5$ hours, the probability of successful islanding drops below 60%. For $\kappa = 10$ hours, the probability level is below 5%.

We do not include in the figure values of κ that are above ten hours, as the p-max problem yields probabilities close to 0%. This means that, with outages longer than 10 hours and with the given MG sizing, the reserves are not enough to achieve a non-zero probability of successful islanding.

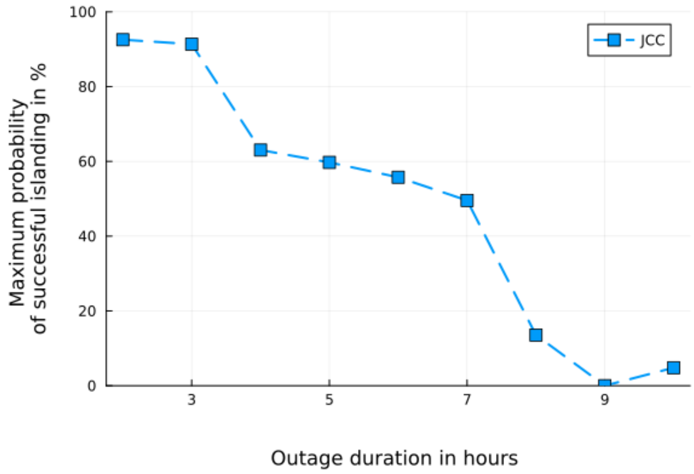

Figure 16: Variation of the maximum probability of successful islanding of the JCC model formulation depending on the outage duration

Overall, we prove that, through using joint chance constraints, a level of successful islanding probability can be preset and guaranteed, increasing the robustness of the optimal MG operation strategy under uncertainty, in exchange with a small drop in profitability. This trade-off is quickly compensated for, given the high reliability level that is achieved.

Practically, when applying dispatch models to the real-time operation of mini-grids, operators implement the dispatch algorithms in a rolling horizon method, typically of 15 minutes, thus updating the operation at each new run with new input data and forecast values. This method should also be applied when using the presented JCC model, as this increases the practical dispatch's adaptability to new boundary conditions.

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