1. Let $\mathcal{X}$ be a finite set, $H : \mathcal{X} \to \mathbb{R}$ and

$$
\mu_\beta(x) := \frac{1}{Z_\beta} e^{-\beta H(x)},
Z_\beta := \sum_{x \in \mathcal{X}} e^{-\beta H(x)}.
$$

This is the simplest example of a Gibbs measure corresponding to the “Hamiltonian” or “energy function” $H$, with inverse thermodynamic temperature $\beta = 1/(k_B T)$, and $k_B$ is the Boltzmann constant. Let $W \subset \mathcal{X}$, and define for any $A \subset \mathcal{X}$,

$$
\mu_\beta^W (A) := \mu_n(A|W) = \operatorname{Prob}(X_\beta \in A | X_\beta \in W).
$$

What is the large-deviation rate function $I^W : \mathcal{X} \to [0, \infty]$ corresponding to the conditioned measure $\mu_\beta^W$, in the low-temperature regime $\beta \to \infty$?

Validity check: is $\inf I^W = 0$?