In exercise sheet 1 we almost proved the following essential tool.

**Theorem 0.1** (Contraction principle). Assume that the random variables $X^n$ satisfy an LDP in a topological space $X$ with good rate functional $I : X \to [0, \infty]$. Define $Y^n := \phi(X^n)$ where $\phi : X \to Y$ is a continuous mapping between the two topological spaces $X$ and $Y$. Then $Y^n$ satisfies an LDP with good rate functional $J : Y \to [0, \infty]$, $$J(y) := \inf_{x \in X : \phi(x) = y} I(x).$$

*Note:* This does not work without the goodness, since $\phi$ maps compact sets to compact sets, but only the pre-image of a closed set is closed!

1. Let $X_1, X_2, \ldots$ be iid random variables in $[-1, 1]$ with Law($X_1$) = $\nu$, and let $S_n := \sum_{i=1}^n X_i$ be the empirical sum. Use Sanov’s Theorem to prove that $\frac{1}{n} S_n$ satisfies an LDP in $[-1, 1]$, with an expression of the rate functional that differs from the one from Cramér’s Theorem. (There is no need to prove that the two rate functionals are the same; this follows from uniqueness of large deviation rate functionals!)