Prof. Alexander Mielke, mielke@wias-berlin.de
10th of October 2022

Selected topics in applied analysis: gradient systems and their gradient flows

Lecture times (may be changed upon request of BMS)
Friday 9:15–10:45 h, Rudower Chaussee 25, Room 2.006
Friday 11:15–12:45 h, Rudower Chaussee 25, Room 2.006
(The excersises will be integrated in the course and not held on special dates. Typically we will meet 90+90 minutes on one day, but there will be days without lectures depending on my personal traveling.)

Starting date: Friday, October 21, 2022

Office hours: Friday 14:00-15:00 h in Room 2.104 (RUD 25)
and after special arrangement (via phone/e-mail) at WIAS

Prerequisites:
necessary: Analysis I–III, Linear functional analysis, linear partial differential equations
recommended: Direct method in the calculus of variations

The course consists of classical blackboard lectures, hopefully with many discussions with the students. My plan is to create a (LaTeX) script of the course, however, this will only be available one or two weeks later. Hence, taking notes is highly recommended.

Planned Topics:
1. Introduction to gradient systems and motivation
2. Gradient systems based on Hilbert spaces
3. Generalized gradient systems in Banach spaces (via EDP)
4. Gradient systems in metric spaces (via EVI)
5. Evolutionary Γ-convergence

Literature

- Modeling with and of gradient systems: [Ott96, Mie11, Pel14]
- Surveys on gradient systems: [San17, Pel14, ChF10]
- Analysis of gradient systems: [Bré73, CoV90, Ott01, AGS05, MRS13, Mie16, MuS20]

[AGS05] L. Ambrosio, N. Gigli, and G. Savaré. *Gradient flows in metric spaces and in the space of probability measures.* Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2005.

[Bré73] H. Brézis. *Opérateurs maximaux monotones et semi-groupes de contractions dans les espaces de Hilbert.* North-Holland Publishing Co., Amsterdam, 1973.

[ChF10] R. Chill and E. Fašangová. *Gradient systems.* matfyzpress, Charles University Prague, 2010.

[CoV90] P. Colli and A. Visintin. On a class of doubly nonlinear evolution equations. *Comm. Partial Differ. Eqns.*, 15(5), 737–756, 1990.

[Mie11] A. Mielke. A gradient structure for reaction-diffusion systems and for energy-drift-diffusion systems. *Nonlinearity*, 24, 1329–1346, 2011.

[Mie16] A. Mielke. On evolutionary Γ-convergence for gradient systems (Ch. 3). In A. Muntean, J. Rademacher, and A. Zagaris, editors, *Macroscopic and Large Scale Phenomena: Coarse Graining, Mean Field Limits and Ergodicity*, Lecture Notes in Applied Math. Mechanics Vol. 3, pages 187–249. Springer, 2016. Proc. of Summer School in Twente University, June 2012.

[MRS13] A. Mielke, R. Rossi, and G. Savaré. Nonsmooth analysis of doubly nonlinear evolution equations. *Calc. Var. Part. Diff. Eqns.*, 46(1-2), 253–310, 2013.

[MuS20] M. Muratori and G. Savaré. Gradient flows and evolution variational inequalities in metric spaces. I: structural properties. *J. Funct. Analysis*, 278(4), 108347/1–67, 2020.

[Ott96] F. Otto. Double degenerate diffusion equations as steepest descent. Preprint no. 480, SFB 256, University of Bonn, 1996.

[Ott01] F. Otto. The geometry of dissipative evolution equations: the porous medium equation. *Comm. Partial Diff. Eqns.*, 26, 101–174, 2001.

[Pel14] M. A. Peletier. Variational modelling: Energies, gradient flows, and large deviations. arXiv:1402.1990, 2014.

[San17] F. Santambrogio. {Euclidean, metric, Wasserstein} gradient flows: an overview. *Bull. Math. Sci.*, 7(1), 87–154, 2017.