Achieving Non-Zero Information Velocity in Wireless Networks

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Abstract

In wireless networks, where each node transmits independently of other nodes in the network (the ALOHA protocol), the expected delay experienced by a packet until it is successfully received at any other node is known to be infinite for signal-to-interference-plus-noise-ratio (SINR) model with node locations distributed according to a Poisson point process. Consequently, the information velocity, defined as the limit of the ratio of the distance to the destination and the time taken for a packet to successfully reach the destination over multiple hops, is zero, as the distance tends to infinity. A nearest neighbor distance based power control policy is proposed to show that the expected delay required for a packet to be successfully received at the nearest neighbor can be made finite. Moreover, the information velocity is also shown to be non-zero with the proposed power control policy. The condition under which these results hold does not depend on the intensity of the underlying Poisson point process.

I. Introduction

Typically, nodes in a wireless network are separated by large distances and packets are routed from source to their destination via many other nodes or over multiple hops. Therefore, to understand the connectivity or information flow in a wireless network, a space-time SINR graph is considered. Such a graph models the evolution of the spatial as well as the temporal connections in the network. The

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space-time SINR graph is a directed and weighted multigraph that represents the most complete random graph model for wireless networks [1]. The SINR (signal-to-interference-plus-noise-ratio) is a ratio of the relative strength of the intended signal and the undesirable interference from simultaneously active unintended nodes of the wireless network. The SINR between any two nodes evolves with time and depends not only on the distance between the two nodes but also on the location of the other nodes in the network. At any time, a directional connection is established from a node at x to another node at y if the SINR from x to y is larger than a threshold. Such a connection represents the ability of node x to deliver meaningful information to y.

Let $\Phi \subset \mathbb{R}^2$ be a point process that specifies the location of the nodes of the network. For any $t \in \mathbb{Z}_+$, let $\Phi_T(t) \subset \Phi$ be the set of nodes that are transmitting at time t and $\Phi_R(t) = \Phi \setminus \Phi_T(t)$ be the set of nodes in receiving mode at time t. Formally, the SINR from a node $x \in \Phi_T(t)$ to a node $y \in \Phi_R(t)$ is given by

$$\mathsf{SINR}_{xy}(t) := \frac{P_x(t) h_t(x,y) \ell(x,y)}{\gamma \sum_{z \in \Phi_T(t) \setminus \{x,y\}} P_z(t) h_t(z,y) \ell(z,y) + \mathsf{N}}, \tag{1}$$

where $\ell(.,.)$ is the distance based signal attenuation or *path-loss* function, $P_x(t)$ is the transmitted power from x at time t, γ is the interference suppression constant, $h_t(u,v)$, $u, v \in \Phi$, are the space-time fading coefficients that model the loss (or gain) from node u to v at time t due to signal propagation via a wireless medium, and N is the variance of the so-called additive white Gaussian noise. By an abuse of notation, we will often use $\ell(|x-y|)$ for $\ell(x,y)$, since the path loss is a function of the distance between x and y. The term $\sum_{z \in \Phi_T(t) \setminus \{x,y\}} P_t(z) h_t(z,y) \ell(z,y)$ in the denominator of (1) is referred to as the interference. Note that we do not include the nodes at x, y in the interference term since transmission from node x is the signal of interest and node y is in receiving mode. $\mathsf{SINR}_{xy}(t)$ is set to be zero at time t if either node $x \in \Phi_R(t)$ or if node $y \in \Phi_T(t)$. Define the indicator random variables

$$e_{xy}(t) := \begin{cases} 1 & \text{if } \mathsf{SINR}_{xy}(t) > \beta, \\ 0 & \text{otherwise}, \end{cases}, \tag{2}$$

where $\beta > 0$ is arbitrary. The space-time SINR graph is defined to be the graph $(\Phi \times \mathbb{Z}_+, E)$, where a directed edge exists from (x, t) to $(y, t+1)$ if $e_{xy}(t) = 1$. Given Φ, the location of the nodes is static, and the time evolution of the graph is entirely due to changes in the fading variables $h_t(u, v)$ and the set Φ_T.

In this paper, we consider a space-time SINR graph in which the location of the nodes is modeled as a homogeneous Poisson point process (PPP). Modeling location of nodes in a wireless networks as a PPP is quite attractive from an analytical point of view and has paid rich dividends in terms of finding exact expressions for several performance indicators such as maximum rate of transmission (capacity), connection probability, etc. [2]–[5], that are hard to derive otherwise. A PPP node location model is well suited for modeling both the ad hoc networks, where large number of nodes are located in a large area without any coordination, as well as the modern paradigm of cellular networks [5], where multiple different layers of base-stations (BSs) (macro, femto, pico) are overlaid on top of each other, and the union of all BSs appears to be uniformly distributed.

Given Φ, the stochastic nature of the fading coefficients $h_t(\cdot, \cdot)$ and the set $\Phi_T(\cdot)$ implies that the event $e_{xy}(t)$ is a random variable, and hence potentially, multiple transmissions are required for successfully transmitting a packet from node x to y. Repeated transmissions entail *delay* in packet transmission, and it is of interest to make the expected delay as small as possible. Another related quantity of interest is the *information velocity*, that is defined as the limit of the ratio of the distance between the source and the destination of any packet, to the total delay experienced by the packet to reach its destination successfully over multi-hops, as distance goes to infinity.

Expected delay and the information velocity are closely connected to the various notions of capacities in wireless networks, e.g., throughput capacity [2], transport capacity [2], delay-normalized transmission capacity [6], [7] etc., since all of them are measures based on the successful rate of departure of packets towards their destination. Finding the speed of information propagation is also related to first passage percolation [8], [9], and dynamic epidemic processes [10]–[12], however, the analysis in the space-time SINR graph gets complicated due to the presence of interference.

In the seminal paper [2], it was shown that with PPP distributed node locations (albeit for a somewhat simpler model), the per-node throughput (rate of transmission between any two randomly selected nodes) or information velocity tends to zero as the size of the network grows. The most general analysis on the expected delay and the information velocity has been carried out in [13] for a PPP-driven space-time SINR network. It is shown that with an ALOHA protocol, where nodes transmit independently of all other nodes with fixed power, the expected delay required for a packet to leave a given node and be successfully received at any other node in the network is infinite. Remarkably, this result is shown to hold even in the absence of interference and requires only the additive noise to be present. Moreover, the information velocity is also shown to be zero. These result have tremendous 'negative' impact on network design, since it shows that essentially any packet cannot exit its source with finite expected delay.

Both the results from [13], viz., the infinite expected delay and zero information velocity, are attributed to the fact that with PPP distributed node locations, a typical node can have large voids around itself, that is, regions that contain no other nodes with high probability. In such a circumstance, a large number of retransmission attempts will be required to overcome the effect of additive noise and support a minimum SNR at any of the other receiving nodes. Consequently, the mean exit delay is infinite (when averaged over the realizations of the PPP) and the information velocity tends to zero.

One solution prescribed in [13] to make the information velocity non-zero is to add another regular square grid of nodes with a fixed density, in which case the nearest neighbor distance is bounded, and the information velocity can be shown to be non-zero. From a practical point of view it is rather limiting to assume the presence of such a regular grid.

Some work has been reported on finite expected delay together with a finite bound on the information velocity [1], [14], [15], under restrictive assumptions such as assuming temporal independence with the SINR model, i.e. interference is independent between any two nodes over time, no power constraint and temporal interference independence, and no additive white Gaussian noise, respectively.

In this paper, we propose a power control mechanism to show that the information velocity can be made non-zero for the space-time SINR graph with PPP node locations without any additional restrictive assumptions on the network. In [13], the information velocity is defined as the limit of the ratio of the distance between two points x and y to the time it takes for the packet to go from x to y as the distance tends to infinity. The packet simultaneously traverses multiple paths and the time taken is the first time the packet is received at y. This makes the set-up somewhat complicated to work with and so the results in [13] are proved for delays averaged over the fading variables. In order to overcome this problem, we define the information velocity somewhat differently, the precise definition of which will appear later. We track a tagged packet as it traverses the network via a *conic* forwarding strategy along a prescribed path. The velocity of this tagged particle along the prescribed path is what we will refer to as the information velocity. Briefly stated, conic forwarding works as follows. At each node, the $\mathbb{R}^2$ plane is partitioned into multiple cones, and each node transmits the packet at the head of its queue to its nearest neighbor in the cone that contains the packet's destination until it is successfully received. We refer to such a cone as the destination cone. This conic forwarding idea circumvents the problem of forming nearest neighbor loops, since the packet always progresses towards its destination. This also allows us to exploit the various independences that exist across space and time.

The aim of the power control strategy is to nullify the path-loss from a node towards its nearest neighbor in the destination cone. In particular, since the path-loss between node x and its nearest neighbor $n(x)$ in the destination cone is $\ell(|x - n(x)|)$, the transmitted power P is taken to be $c\ell(|x - n(x)|)^{-1}$, where c is a constant. To compensate for the non-homogeneity in power used at each node, we modulate the transmission probability p, such that pP equals than the average power constraint at each node.

We wish to note that per se, power control and conic forwarding are not new concepts in wireless communication. For instance, power control is used by most mobiles phones as part of current wireless technology standards. However, its use and advantages in large wireless networks with randomly located nodes has not been explored sufficiently.

Using conic forwarding strategy together with power control, we show that the expected delay to the nearest neighbor in the destination cone is finite provided $\beta\gamma < 1$. In addition, as the tagged packet traverses the network from one node to another, we add additional nodes in the network. This increases the interference but delivers a stationary sequence of delay times. This enables us to apply the ergodic theorem and infer that the information velocity is strictly positive with probability one. For the special case when the path loss has bounded support, we show that the information velocity is bounded below by a strictly positive constant.

II. System Model

Let Φ be a homogenous PPP with intensity λ in $\mathbb{R}^2$ modeling the location of the nodes of the network. The time parameter is assumed to be discrete (slotted). Let $\{h_t(x,y), x, y \in \Phi, t = 0, 1, \ldots\}$ be a collection of independent exponentially distributed random variables with parameter μ. $h_t(x,y)$ is the fading power from node x to node y in the time slot t. The path loss between $x, y \in \Phi$ denoted by $\ell(x,y) = \ell(|x-y|)$ is given by

$$\ell(r) = r^{-\alpha} \wedge 1, \qquad r > 0,$$

where $a \wedge b = \min(a,b)$ and $\alpha > 2$ is arbitrary.

We assume that each node can only operate in a half-duplex mode, that is, in the time slot t, a node $x \in \Phi$ is *on* (transmitter) or *off* (receiver) following a Bernoulli random variable $\mathbf{1}_x(t)$, with $\mathbb{P}(\mathbf{1}_x(t) = 1) = p_x(t)$. Let $q_x(t) = 1 - p_x(t)$. The set of *on* (*off*) nodes in the time slot t is denoted by $\Phi_T(t)$ ($\Phi_R(t)$).

Let $C_1, \ldots, C_m$ be cones with angle $2\phi < \frac{\pi}{2}$ in $\mathbb{R}^2$ with vertex at the origin, satisfying $\cup_{i=1}^m C_i = \mathbb{R}^2$ and $C_i \cap C_j = \emptyset$ for $i \neq j$, as shown in Fig. 1. Without loss of generality, suppose that C_1 is symmetric about the x-axis and opens to the right. Let $x + C_i$ be translation of cone C_i by x. In the time slot t, for a node x, let $x + C_d(x,t)$ be the cone that contains the final destination of the packet that it wishes to transmit. We call this cone as the destination cone. Denote the nearest neighbor of x in the destination cone $x + C_d(x,t)$ by $n_t(x)$. If the node at x is *on* in time slot t then it transmits with power $P_x(t)$. The key idea in this paper is to employ a decentralized power control scheme, that is, the functions $p_x(t), Px(t)$

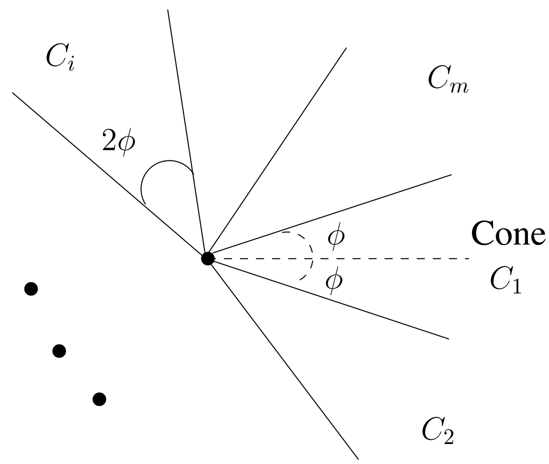

Fig. 1. Definition of cones with angle 2ϕ.

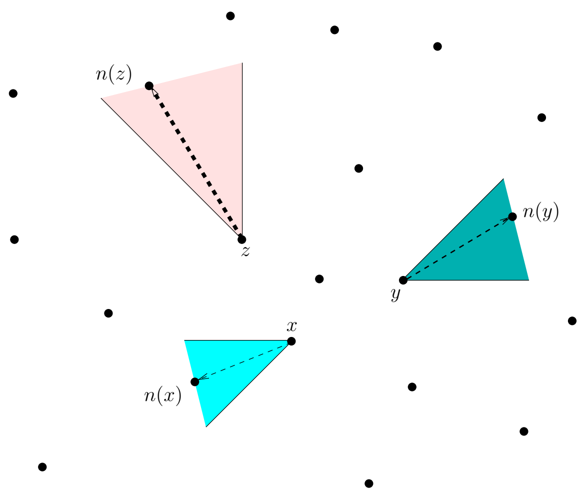

Fig. 2. Each node transmits to its nearest neighbor in the destination (shaded) cone.

depend locally on Φ. The particular forms that these functions take are given by

$$P_x(t) = c\ell^{-1}(x, n_t(x)), \qquad p_x(t) = M(P_x(t))^{-1}, \tag{3}$$

where $c = M(1-\epsilon)^{-1}$, $0 < \epsilon < 1$ is a constant, and $M = P_x(t)p_x(t)$ is the average power constraint. Note that $p_x(t) \leq 1-\epsilon$, since $\ell(\cdot) \leq 1$. Thus, in each time slot, each node makes transmission attempts with transmission power proportional to the distance to its nearest neighbor in the destination cone to compensate for the path-loss to the nearest neighbor. The transmission probability is chosen so as to satisfy an average power constraint.

In Fig. 2, we illustrate the transmission strategy, where each node transmits to its nearest neighbor in the destination cone (shaded cone) with line thickness proportional to the transmit power, farther the nearest neighbor, larger the power. In prior work [13], with the ALOHA protocol, the functions P and p were assumed to be constants that were independent of the system parameters.

Thus, the SINR from node x to node y in time slot t is given by

$$\mathsf{SINR}_{xy}(t) = \frac{P_x(t)h_t(x,y)\ell(x,y)\mathbf{1}_x(t)(1-\mathbf{1}_y(t))}{\gamma\sum_{z\in\Phi_T(t)\setminus\{x,y\}} P_z(t)h_t(z,y)\ell(z,y)+\mathsf{N}}, \tag{4}$$

where $0 < \gamma < 1$ is the processing gain of the system (interference suppression parameter) which depends on the transmission/ detection strategy, for example, on the orthogonality between codes used by different legitimate nodes during simultaneous transmissions. The transmission from node x to y is deemed successful at time t, if $\mathsf{SINR}_{xy}(t) > \beta$, where $\beta > 0$ is a fixed threshold. Let $e_{xy}(t) = 1$ if $\mathsf{SINR}_{xy}(t) > \beta$, and zero otherwise. The sum in the denominator of the right hand expression in the above equation is referred to as the interference and $\alpha > 2$ ensures that the interference term in the denominator is finite almost surely. Since $h_t(x,y)$ is exponentially distributed, multiple transmissions may be required for a packet to be successfully received at any node.

III. Main Results and Proofs

Our first objective is to show that with the power control policy described above, the expected time for a packet to be successfully received at the nearest neighbor in the destination cone is finite.

Definition 3.1: Let the minimum time (exit time) taken by any packet to be successfully transmitted from node x to its nearest neighbor $n(x)$ in the destination cone of the packet be

$$T(x) = \min\left\{t > 0 : e_{x,n_t(x)}(t) = 1\right\}.$$

Theorem 3.2: [Finite expected exit time] Suppose $\beta\gamma < 1$. Then for all $\epsilon > 0$ sufficiently small, the space-time SINR graph with power control policy as described above satisfies $\mathbb{E}\{T(x)\} < \infty$ for any $x \in \Phi$.

Remark 3.3: The parameter β controls per link data rate, larger the value of β, larger is the per link data rate. The condition $\beta\gamma < 1$ indicates that to support larger per-link data rate, one has to invest in getting a better (lower) interference suppression parameter, e.g. by lowering the chip rate in a wireless CDMA system. The condition $\beta\gamma < 1$ also indicates that there is no free lunch, if one wants larger data rate and finite expected exit time, one has to have better interference suppression capability. An interesting upshot of the proposed the power control policy is that the condition required for the theorem to hold is independent of the intensity λ of the PPP.

Proof: Without loss of generality, suppose that the origin $o \in \Phi$. We tag a particular packet to be transmitted out of the node at o, and suppose that the destination cone of this packet when it is at o is C_d. Denote the nearest neighbor of o in C_d by $n(o)$. We have dropped the time subscript from $n(o)$, since, as long as the packet is not successfully transmitted out of o the destination cone remains the same. Let

$$\mathsf{SINR}_{o,n(o)}(t) = \frac{P_o(t)h_t(o,n(o))\ell(o,n(o))\mathbf{1}_o(t)(1-\mathbf{1}_{n(o)}(t))}{\gamma I(t) + \mathsf{N}}, \tag{5}$$

where

$$I(t) = \sum_{z \in \Phi \setminus \{o,n(o)\}} \mathbf{1}_z(t)P_z(t)h_t(z,n(o))\ell(z,n(o)). \tag{6}$$

Let $e_{o,n(o)}(t) = 1$ if $\mathsf{SINR}_{o,n(o)}(t) > \beta$, and 0 otherwise. Due to interference and the nature of the traffic arriving at the nodes, the choice of the destination cones are not independent across time slots at the same node as well as at different nodes. Hence to evaluate $\mathbb{E}\{T(o)\}$ we need to condition appropriately. Let $\mathcal{G}_k$ be the sigma field generated by the point process Φ and the choice of cones made at all nodes of Φ at times $t = 1, 2, \ldots, k$. Note that as long as the packet is not successfully transmitted out of o, the transmission probability $p_o(t)$ does not change. Let $F := \cap_{j=2}^{k}\{p_o(j) = p_o(1)\}$. Now

$$\mathbb{P}\left[T(o) > k \middle| \Phi\right] = \mathbb{E}\left\{\mathbb{P}\left[e^d_{o,n(o)}(t) = 0,\ \forall\, t = 1, \ldots, k \middle| \mathcal{G}_k\right] \mathbf{1}_F \middle| \Phi\right\}.$$

Let $A(t)$ be the event that the origin $o \in \Phi_R(t)$, and $B(t)$ be the event that $o \in \Phi_T(t)$, $n(o) \in \Phi_R(t)$ but $\mathsf{SINR}_{o,n(o)}(t) \leq \beta$. Writing the expression on the right above in terms of $A(t), B(t)$, and using the independence of the fading powers and the conditional independence of the transmission events, we get

$$\mathbb{P}\left[T(o) > k \middle| \Phi\right] = \mathbb{E}\left\{\prod_{t=1}^{k} \mathbb{P}\left[A(t) \cup B(t) \middle| \mathcal{G}_k\right] \mathbf{1}_F \middle| \Phi\right\}. \tag{7}$$

On the event F, we have $\mathbb{P}(A(t)|\mathcal{G}_k) = 1 - p_o(1)$ and

$$\mathbb{P}(B(t)|\mathcal{G}_k) = p_o(1)q_{n(o)}(t)\left(1 - \mathbb{E}\left\{\exp\left(-\frac{\mu\beta}{c}(\mathsf{N} + \gamma I(t))\right)\Big|\mathcal{G}_k\right\}\right), \tag{8}$$

for $t = 1, 2, \ldots, k$. (8) follows from (5) by using the fact that $P_o(t)\ell(o, n(o)) = c$ and taking expectation with respect to $h_t(o, n(o)) \sim \exp(\mu)$. This yields

$$\begin{aligned}\mathbb{P}\left[A(t) \cup B(t)\big|\mathcal{G}_k\right] &\leq 1 - p_o(1) + p_o(1)q_{n(o)}(t)\left(1 - \mathbb{E}\left\{e^{-\frac{\mu\beta}{c}(\mathsf{N}+\gamma I(t))}\Big|\mathcal{G}_k\right\}\right) \\ &= 1 - p_o(1)p_{n(o)}(1) - p_o(1)q_{n(o)}(t)e^{-\frac{\mu\beta\mathsf{N}}{c}}\mathbb{E}\left\{e^{-\frac{\mu\beta\gamma}{c}I(t)}\Big|\mathcal{G}_k\right\} \\ &\leq 1 - p_o(1)\,\epsilon\, e^{-\frac{\mu\beta\mathsf{N}}{c}}\,\mathbb{E}\left\{e^{-\frac{\mu\beta\gamma}{c}I(t)}|\mathcal{G}_k\right\},\end{aligned} \tag{9}$$

where we have used the fact that $q_y(t) \geq \epsilon$. Let $a = \frac{\mu\beta\gamma}{c}$. We now find a lower bound for the expectation on the right hand side above that is independent of the choice of the cone. To this end observe that

$$\mathbb{E}\left\{e^{-\frac{\mu\beta}{c}I(t)}\Big|\mathcal{G}_k\right\} = \prod_{z\in\Phi\setminus\{o,n(o)\}}\mathbb{E}\left\{e^{-a\mathbf{1}_z(t)P_z^{(i)}(t)h_t(z,n(o))\ell(z,n(o))}\Big|\mathcal{G}_k\right\} \tag{10}$$

Suppose node $z \in \Phi\setminus\{o, n(o)\}$ transmits using cone $z + C_i$ in time slot t. This fixes the transmission probability $p_z^{(i)}(t)$ and power $P_z^{(i)}(t)$ (where we have included the index i to make the dependence on the cone explicit). Then

$$\begin{aligned}&\mathbb{E}\left\{e^{-\frac{\mu\beta\gamma}{c}\mathbf{1}_z(t)P_z^{(i)}(t)h_t(z,n(o))\ell(z,n(o))}\Big|\mathcal{G}_k\right\} \\ &\quad= (1 - p_z^{(i)}(t)) + p_z^{(i)}(t)\mathbb{E}\left\{e^{-aP_z^{(i)}(t)h_t(z,n(o))\ell(z,n(o))}\Big|\Phi\right\} \\ &\quad= (1 - p_z^{(i)}(t)) + p_z^{(i)}(t)\frac{c}{c + \beta\gamma\ell(z, n(o))P_z^{(i)}(t)}.\end{aligned} \tag{11}$$

Let $C_z^* = C_z^*(\Phi)$ be the cone for which the right hand expression in (11) is minimized. Let p_z^*, P_z^* denote the corresponding transmission probability and power respectively. Denote by $\mathbf{1}_z^*$ an independent Bernoulli random variable with $\mathbb{P}[\mathbf{1}_z^* = 1] = p_z^*$. The cone C_z^* maximizes the interference contribution at $n(o)$ due to transmission at z. Define

$$I^*(t) = I^*(t, \Phi) = \sum_{z\in\Phi\setminus\{o,n(o)\}}\mathbf{1}_z^*P_z^*h_t(z, n(o))\ell(z, n(o)). \tag{12}$$

Substituting $I^*(t)$ for I in (9) along with the observation that given Φ, $I^*(t) \stackrel{d}{=} I^*(1)$ we get

$$\mathbb{P}\left[A(t) \cup B(t) \middle| \mathcal{G}_k\right] \leq 1 - p_o(1)\, \epsilon\, e^{-\frac{\mu\beta \mathsf{N}}{c}}\, \mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\}. \tag{13}$$

Substituting from (13) in (7), we get

$$\mathbb{P}\left[T(o) > k \middle| \Phi\right] \leq (1-J)^k, \tag{14}$$

where

$$J = p_o(1)\epsilon e^{-\frac{\mu\beta \mathsf{N}}{c}} \mathbb{E}\left\{\exp\left(-aI^*(1)\right) \middle| \Phi\right\}.$$

The expected delay can then be written as

$$\begin{aligned} \mathbb{E}\{T(o)\} &= \sum_{k\geq 0} \mathbb{P}[T(o) > k] \\ &= \mathbb{E}\left\{\sum_{k\geq 0} \mathbb{P}\left[T(o) > k \middle| \Phi\right]\right\} \\ &\leq \mathbb{E}\{J^{-1}\}. \end{aligned}$$

By the Cauchy-Schwartz inequality we get

$$\mathbb{E}\{T(o)\} \leq \frac{e^{\frac{\mu\beta \mathsf{N}}{c}}}{\epsilon} \left(\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\}\right)^2}\right\} \mathbb{E}\left\{p_o(1)^{-2}\right\}\right)^{\frac{1}{2}}. \tag{15}$$

From the definition of the transmission probability $p_o(t)$, we get

$$\mathbb{E}[p_o(1)^{-2}] \leq \mathbb{E}\left\{\left(\frac{c}{M}\right)^2 \left(|n(o)|^{2\alpha} \vee 1\right)\right\} < \infty, \tag{16}$$

since the nearest neighbor distance in a cone has density

$$f(r) = \frac{2\lambda\pi r}{m} e^{-\frac{\lambda\pi}{m} r^2}, \qquad r > 0. \tag{17}$$

It remains to show that

$$\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\}\right)^2}\right\} < \infty. \tag{18}$$

$$\mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\} = \prod_{z \in \Phi \setminus \{o, n(o)\}} \mathbb{E}\left\{e^{-a\mathbf{1}_z^* P_z^* h_1(z,n(o))\ell(z,n(o))} \middle| \Phi\right\}. \tag{19}$$

Taking expectations, first with respect to $\mathbf{1}_z^*$ and then with respect to $h_1(z, n(o))$, we get

$$\begin{aligned}
\mathbb{E}\left\{e^{-a\mathbf{1}_z^* P_z^* h_1(z,n(o))\ell(z,n(o))} \middle| \Phi\right\} &= (1-p_z^*) + p_z^* \mathbb{E}\left\{e^{-aP_z^* h_1(z,n(o))\ell(z,n(o))} \middle| \Phi\right\} \\
&= 1 - p_z^* \left(1 - \frac{\mu}{\mu + aP_z^* \ell(z, n(o))}\right) \\
&\overset{(a)}{=} 1 - \frac{\beta\gamma p_z^* P_z^* \ell(z,n(o))}{c + \beta\gamma P_z^* \ell(z,n(o))} \\
&\overset{(b)}{\geq} 1 - \frac{\beta\gamma M \ell(z,n(o))}{c} = 1 - \beta\gamma(1-\epsilon)\ell(z,n(o)),
\end{aligned}$$

where (a) follows by substituting $\frac{\mu\beta\gamma}{c}$ for a, and to obtain (b) we have used the fact that the average power $p_z P_z$ equals M and $c = M(1-\epsilon)^{-1}$. Let $c_1 = \beta\gamma(1-\epsilon)$. Substituting the above bound in (19) we get

$$\mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\} \geq \prod_{z \in \Phi \setminus \{o, n(o)\}} \left(1 - c_1 \ell(z, n(o))\right). \tag{20}$$

Note that $c_1 \ell(z, n(o)) < 1$ since $\beta\gamma < 1$. Let $B(x, r)$ denote a ball of radius r centered at x. Substituting the bound obtained in (20) in (18), we get

$$\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\}\right)^2}\right\} \leq \mathbb{E}\left\{\prod_{z \in (\Phi \setminus \{o,n(o)\}) \cup \Phi_0} e^{-2\log(1 - c_1 \ell(|z|))}\right\}, \tag{21}$$

where the last inequality follows by shifting the origin to $n(o)$ and including points from an independent Poisson process Φ_0 of intensity $\lambda \mathbf{1}_{\{(o+C_d) \cap B(o, |n(o)|)\}}$, i.e., a PPP of intensity λ restricted to the set $(o + C_d) \cap B(o, |n(o)|)$. Clearly $(\Phi \setminus \{o, n(o)\}) \cup \Phi_0$ is a PPP of intensity λ with the origin at $n(o)$. Hence by an application of the Campbell's theorem in (21) and the fact that $\ell(|z|) \leq 1$ we get

$$\begin{aligned}
\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-aI^*(1)} \middle| \Phi\right\}\right)^2}\right\} &\leq \exp\left(\lambda \int_{\mathbb{R}^2} \left(e^{-2\log(1 - c_1\ell(|z|))} - 1\right) \mathrm{d}z\right) \\
&\leq \exp\left(\frac{2\lambda c_1}{(1-c_1)^2} \int_{\mathbb{R}^2} \ell(|z|) \mathrm{d}z\right) < \infty,
\end{aligned} \tag{22}$$

since $\alpha > 2$. This completes the proof of Theorem 3.2. ■

Next, we build upon Theorem 3.2, to show that the information velocity, that is, the rate at which packets flow towards their destination, is positive under the proposed power control mechanism. Information velocity is a key quantity in multi-hop routing. Larger the velocity, higher is the capacity of the network. The negative results in [13] on the infinite expected delay and zero information velocity are proved for delays that are averaged over the fading variables. In order to work with the delay variables directly and also to be able to use the ergodic theorem, we introduce several additional structures as we go along.

As a first step we track the movement of a tagged packet that starts at the origin $X_0 = o \in \Phi$ and traverses the network as follows. Let T_0 be the time taken by this tagged packet starting at the origin to successfully reach its nearest neighbor $X_1 = n(o)$ in the destination cone C_1. The packet is transmitted with power $P_1 = c\ell(|n(o))|)^{-1}$ and the probability that it is transmitted in a time slot is MP_1^{-1}. Going forward, if the packet is at node X_{i-1}, $i \geq 2$, let T_{i-1} be the time taken for the packet to successfully reach the nearest neighbor X_i of X_{i-1} in the destination cone $X_{i-1}+C_1$. From the time the packet reaches X_{i-1} to the time it is successfully delivered at X_i, it is transmitted with power $P_i = c\ell(|X_i - X_{i-1})|)^{-1}$ and transmission probability MP_i^{-1}. One can think of the destination of the packet being located at $(\infty, 0)$ and thus the destination cone for this packet is always a translation of the cone C_1. We ignore the queuing and other delays at each node as is the standard practice. Note that the delays T_i, $i \geq 0$, are not identically distributed. For instance, the point process as seen from the origin and from $n(o)$ do not have the same distribution. In particular, $\{T_i, i \geq 0\}$ is not a stationary sequence.

Definition 3.4: The information velocity of space-time SINR network is defined as

$$v = \liminf_{t \to \infty} \frac{d(t)}{t},$$

where $d(t)$ is the distance of the tagged packet from the origin at time t.

The following is the main result of this paper.

Theorem 3.5: Under the conditions of Theorem 3.2 and the proposed power control strategy, the information velocity $v > 0$, almost surely. If the path loss function has compact support, that is, for some constant $r_0 > 0$, $\ell(r) = 0$ for all $r > r_0$, then there exists a constant $v_0 > 0$ such that $v \geq v_0$ almost surely.

Proof: In order to prove this result, we first dominate the delays $\{T_i, i \geq 0\}$, by a stationary sequence $\{T_i^{'}, i \geq 0\}$, and show that a positive speed can be obtained even with these enhanced delays. This will be done by adding some additional points that will increase the interference and hence the delay. To this

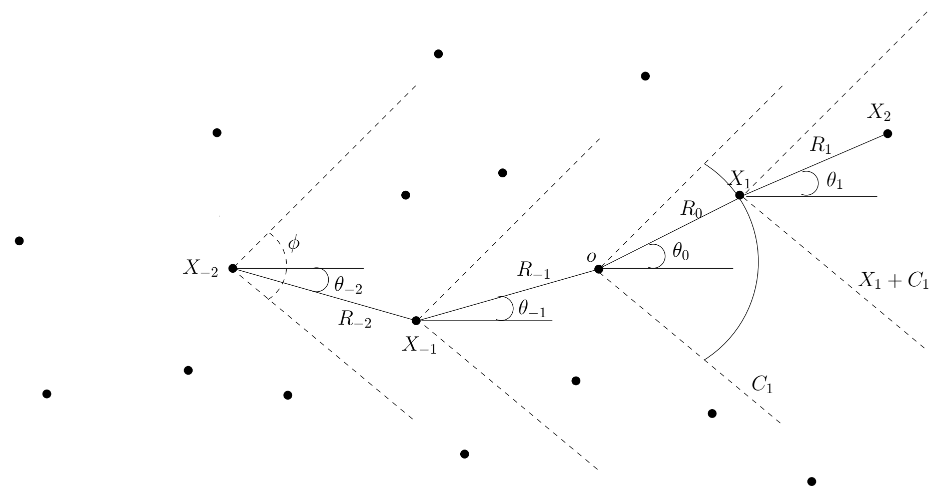

Fig. 3. Illustration of addition on infinite sequence of points/interferers to make T_i' stationary.

end, for all $i \geq 0$, let $R_i = |X_{i+1} - X_i|$ and $\theta_i = \arcsin((X_{i+1,2} - X_{i,2})/R_i)$, where $X_i = (X_{i1}, X_{i2})$. Note that the cones $\{(X_i + C_1) \cap B(X_i, R_i), i \geq 0\}$ are non-overlapping since $\phi < \pi/4$. Consequently, $\{(R_i, \theta_i), i \geq 0\}$ is a sequence of independent and identically distributed random vectors having the same distribution as the random vector (R, θ), where R and θ are independent with density of R given by (17) and θ is uniformly distributed on $(-\phi, \phi)$.

To nullify the effect of moving to the nearest neighbor we progressively fill the voids with independent Poisson points as the packet traverses the network. This however leaves an increasing sequence of special points, the X_i's. The following construction is intended to take care of this issue and deliver a stationary sequence.

Let $\{(R_{-i}, \theta_{-i}, i \geq 1\}$ be a sequence of independent random vectors with each vector having the same distribution as (R, θ). Define $\tilde{\Phi} = \{X_{-i}, i \geq 1\}$, recursively starting from X_{-1} so as to satisfy $|X_{-i} - X_{-i+1}| = R_{-i}$ and $\theta_{-i} = \arcsin((X_{-i+1,2} - X_{-i,2})/R_{-i})$. Observe that each X_{-i+1}, $i \geq 1$, lies in the cone $X_{-i} + C_1$ as shown in Fig. 3, and $\{\Phi \cap ((X_{-i} + C_1) \cap B(X_{-i}, R_{-i})), i \geq 1\}$ is a sequence of independent and identically distributed random variables.

Let T_0' be the delay experienced by the tagged packet in going from X_0 to X_1 when the interference is coming from the points of $(\Phi \setminus \{o, n(o)\}) \cup \tilde{\Phi}$. For $i \geq 0$, let Φ_i be an PPP of intensity $\lambda \mathbf{1}_{\{(X_i + C_1) \cap B(X_i, R_i)\}}$

independent of everything else. For $i \geq 1$, let $T_i^{'}$ be the delay experienced by the tagged particle in going from X_i to X_{i+1} when the interference is coming from the nodes in $(\Phi \setminus \{X_i, X_{i+1}\}) \cup \tilde{\Phi} \cup_{j=0}^{i-1} \Phi_j$. Note that for the actual delay T_i, that is, when the packet is at X_i and trying to reach X_{i+1}, the interference contribution is coming from the nodes in $\Phi \setminus \{X_i, X_{i+1}\}$. To define $T_i^{'}$, we have added additional interferers at $\tilde{\Phi} \cup_{j=0}^{i-1} \Phi_j$. We assume that virtual interferers placed at $\tilde{\Phi} \cup_{j=0}^{i-1} \Phi_j$ behave similar to nodes of Φ. Clearly $T_i^{'} \geq T_i$, and furthermore the sequence $\{T_i^{'}, i \geq 0\}$ is a stationary sequence. To prove the later assertion consider any finite dimensional vector of delays $(T_{i_1}^{'}, T_{i_2}^{'}, \ldots, T_{i_j}^{'})$. The distribution of this vector is a function of the distribution of the special points $\{X_i, i \leq i_j\}$, the point processes $(\Phi \setminus \cup_{i \leq i_j} X_i)$ and $\cup_{i=0}^{i_j-1} \Phi_i$, which by our construction is translation invariant.

Suppose we show that $\eta = \mathbb{E}[T_i^{'}] < \infty$. Then by the Birkoff's ergodic theorem [16], we have

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} T_k^{'} = T^{'},$$

almost surely, where $T^{'}$ is a random variable with mean η.

Let $N(t)$ be the counting process associated with an arrival process with inter-arrival times given by the sequence $\{T_i^{'}, i \geq 0\}$. Then the information velocity satisfies

$$v \geq \lim_{t \to \infty} \frac{\sum_{k=1}^{N(t)} R_k \cos(\theta_k)}{\sum_{k=1}^{N(t)+1} T_k^{'}} = \frac{\mathbb{E}[R \cos(\theta)]}{T^{'}}.$$

The result now follows since $T^{'}$ is finite almost surely.

It remains to show that $E[T_0^{'}] < \infty$. The proof of this assertion proceeds along the same lines as the proof of Theorem 3.2 with $I(t)$ in (6) replaced by $I(t) + \tilde{I}(t)$, where

$$\tilde{I}(t) = \sum_{z \in \tilde{\Phi}} 1_z(t) P_z(t) h_t(z, n(o)) \ell(z, n(o)).$$

This would lead to a bound analogous to (15) with $I^*(1)$ replaced by $I^*(1) + \tilde{I}^*(1)$ and Φ replaced by $\Phi \cup \tilde{\Phi}$, where $\tilde{I}^*(1)$ is defined analogous to $I(1)$. By the conditional independence of $I^*(1)$ and $\tilde{I}^*(1)$ we get

$$\mathbb{E}\left\{ \frac{1}{\left(\mathbb{E}\left\{ e^{-a(I^*(1)+\tilde{I}^*(1))} | \Phi \cup \tilde{\Phi} \right\} \right)^2} \right\} = \mathbb{E}\left\{ \frac{1}{\left(\mathbb{E}\left\{ e^{-aI(1)} | \Phi \right\} \mathbb{E}\left\{ e^{-a\tilde{I}(1)} | \tilde{\Phi} \cup \{n(o)\} \right\} \right)^2} \right\}$$

Another application of the Cauchy-Schwartz inequality implies that the result follows if we show that

$$\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-aI(1)}\middle|\Phi\right\}\right)^4}\right\}\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-a\tilde{I}(1)}\middle|\tilde{\Phi}\cup\{n(o)\}\right\}\right)^4}\right\}<\infty.$$

Proceeding as in (21)-(22), we get

$$\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-aI(1)}\middle|\Phi\right\}\right)^4}\right\}\leq\exp\left(\frac{\lambda}{(1-c_1)^4}\int_{\mathbb{R}^2}\left(1-(1-c_1\ell(|z|))^4\right)\mathrm{d}z\right)<\infty,$$

since $\alpha > 2$. It remains to show that

$$\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-a\tilde{I}(1)}\middle|\tilde{\Phi}\cup\{n(o)\}\right\}\right)^4}\right\}<\infty. \tag{23}$$

To compute the expression in (23) we proceed as we did in (19)-(20) and arrive at the following bound similar to the one in (21).

$$\begin{aligned}\mathbb{E}\left\{\frac{1}{\left(\mathbb{E}\left\{e^{-a\tilde{I}(1)}\middle|\tilde{\Phi}\cup\{n(o)\}\right\}\right)^4}\right\} &\leq \mathbb{E}\left\{\prod_{i=1}^{\infty}e^{-4\log(1-c_1\ell(X_{-i},n(o)))}\right\}\\ &\leq \mathbb{E}\left\{\prod_{i=1}^{\infty}e^{-4\log\left(1-c_1\ell(\sum_{j=0}^{i}R_{-j}\cos(\theta_{-j}))\right)}\right\}\\ &\leq E\left\{e^{\sum_{n=1}^{\infty}g(S_{n+1})}\right\},\end{aligned} \tag{24}$$

where $S_n=\sum_{j=0}^{n-1}R_{-j}\cos(\theta_{-j})$ and $g(x)=-4\log(1-c_1\ell(x))$, $x>0$. Let $\xi=\mathbb{E}\{R\cos(\theta)\}$ and note that $\xi>0$. Let $\delta\in(0,\xi)$ be a constant that will be chosen later. Define $\chi(\nu)=\mathbb{E}\{e^{\nu R\cos(\theta)}\}$, $\nu\in\mathbb{R}$ and let $\zeta(\delta)=\inf\{\nu\delta-\log(\chi(\nu)),\nu>0\}$. That $\chi(\nu)<\infty$ for all ν follows from (17). By the Chernoff bound, we have

$$P\left[\frac{S_n}{n}<\delta\right]\leq e^{-\zeta(\delta)n}.$$

It follows by the Borel-Cantelli lemma that, almost surely, there exists a $N=N(\omega)<\infty$ such that $S_n\geq n\delta$ for all $n\geq N(\omega)$. Hence for some constant $c_2>0$,

$$\begin{aligned}P[N\geq m] &= P[S_n<n\delta \text{ for some } n\geq m],\\ &\leq \sum_{n=m}^{\infty}e^{-\zeta(\delta)n}\leq c_2e^{-\zeta(\delta)m}.\end{aligned} \tag{25}$$

Using the fact that the function g is non-increasing, we get

$$\begin{aligned}\mathbb{E}\left\{e^{\sum_{n=1}^{\infty} g(S_n)}\right\} &= \mathbb{E}\left\{e^{\sum_{n=1}^{N} g(S_n)+\sum_{n=N+1}^{\infty} g(S_n)}\right\}, \\ &\leq e^{\sum_{n=1}^{\infty} g(n\delta)}\mathbb{E}\left\{e^{g(0)N}\right\}.\end{aligned}$$

$\sum_{n=1}^{\infty} g(n\delta) < \infty$ since $\alpha > 2$ by the comparison test. Since $R\cos(\theta) > 0$, $\zeta(\delta) \uparrow \infty$ as $\delta \downarrow 0$. So, we can and do choose δ such that $\zeta(\delta) > g(0)$. With this choice of δ, it follows from (25) that $E\left\{e^{g(0)N}\right\} < \infty$. This proves (23).

If the path loss function has compact support then the sequence $\{T_k^{'}\}_{k\geq 0}$ is also ergodic [16], and consequently $T^{'} \equiv \eta$, and consequently $v \geq \frac{E\{R\cos(\theta)\}}{\eta}$, almost surely. ■

Remark 3.6: By the strong law of large numbers, $\frac{X_n}{n} \to (\mathbb{E}\{R\cos(\theta)\}, 0)$, and thus the asymptotic direction of motion of the tagged packet defined as $\lim_{n\to\infty} \frac{X_n}{|X_n|}$ is indeed $(1, 0)$.

Remark 3.7: The time taken for the tagged packet to perform n-hops or to reach X_n satisfies

$$\limsup_{n\to\infty} \frac{\sum_{i=0}^{n-1} T_i}{n} \leq \eta.$$

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