Mathematical Institute
University Leipzig
Summer term 2005

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ODE for Physicists - written exam

Date: July 9, 2005, 9:00 – 11:00

The only means allowed is a one-sided handwritten sheet of paper containing any text of your choice

last name:
first name:
Matrikelnummer:

Please do not write below this line.

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problem number	1	2	3	4	5	6	7
points							

TOTAL NUMBER OF POINTS:

REACHING 50 POINTS IS SUFFICIENT FOR PASSING SUCCESSFULLY.

GOOD LUCK!

1. (3+10+3+4 POINTS)

   (a) Derive all solutions to the homogeneous ODE $y'' - 6y' + 9y = 0$.

   (b) Use the method of variation of constants to find one particular solution to the ODE $y'' - 6y' + 9y = 2\mathrm{e}^{3x}$.

   (c) What is a correct ansatz of the type of the right hand side for the ODE $y''-6y'+9y = 2\mathrm{e}^{3x}$? (Give the ansatz only; do not carry out the calculation.)

   (d) Find all solutions to the IVP $y'' - 6y' + 9y = 2\mathrm{e}^{3x}, \quad y(0) = 1, y'(0) = 1$.

2. (15 POINTS) Find the general solution to $y' = \frac{y}{x} + (\frac{y}{x})^2$ for $x \in (0, \infty)$.

3. (15 POINTS) It is known that the ODE $xy' = xy^2 + y$ (for $x \in (0, \infty)$) possesses the solution $y_{\mathrm{p}}(x) = \frac{a}{x}$ for some $a \in \mathbb{R} \setminus \{0\}$. What is the general solution?

4. (10 POINTS) Use the method of power series to find the general solution of

$$xy' = y - 1.$$

5. (15 POINTS) Find two functions whose Laplace transforms are the functions

$$Y_1(s) = \frac{1}{(s-2)^3} \qquad \text{respectively} \qquad Y_2(s) = \frac{1}{s^2 - 4}.$$

6. (10 POINTS) Let (X, d) be a metric space and $A \colon X \to X$ a map such that $\mathrm{d}(A^n x, A^n y) \leq \alpha_n \mathrm{d}(x, y)$ for any $x, y \in X$ and any $n \in \mathbb{N}$, where $\alpha_n > 0$ are positive numbers satisfying $\sum_{n \in \mathbb{N}} \alpha_n < \infty$. Prove that A does not possess more than one fixed point in X.

7. (15 POINTS) Find an integrating factor for the equation

$$2(x^2 + x + y) + (2x^2 + 2y + 1)y' = 0.$$