Mathematical Institute
University Leipzig
Summer term 2005

Prof. Dr. Wolfgang König
Dr. Ramon Plaza

ODE for Physicists - Homework 7

Due: May 31, 2005

17. (4 pts.) Let y_1 and y_2 be the two linearly independent solutions to the homogeneous equation $y'' + ay' + by = 0$ that we introduced in the proof of Lemma 3.1.3. Consider the so-called *Wronski determinant*, $W = y_1 y_2' - y_2 y_1'$. Prove that

$$W(x) \neq 0 \qquad \text{and} \qquad W'(x) = -aW(x) \qquad \text{for any } x \in \mathbb{R}.$$

18. (2 pts.) Consider the homogeneous equation $y'' + ay' + by = 0$ with $a, b \in \mathbb{R}$ such that the two solutions $\lambda, \mu \in \mathbb{R}$ of the characteristic equation satisfy $\mu \neq 0$ and $\lambda \neq \pm\mu$. We look at the solutions

$$y_1(x) = \frac{1}{\lambda - \mu}\Big(e^{\lambda x} - e^{\mu x}\Big) \qquad \text{and} \qquad y_2(x) = \frac{1}{\lambda + \mu}\Big(e^{\lambda x} + e^{\mu x}\Big).$$

Prove that

$$\lim_{\lambda \to \mu} y_1(x) = x e^{\mu x} \qquad \text{and} \qquad \lim_{\lambda \to \mu} y_2(x) = \frac{1}{\mu} e^{\mu x}.$$

19. (5 pts.) (a) Find the general solution to $y'' - 2y' + 10y = 0$.

    (b) Solve the IVP $y'' + 6y' + 9y = 0$, $y(0) = 1, y'(0) = -4$.

20. (5 pts.) (a) Find the general solution to $y'' + 4y = x \cos x$.

    (b) Solve the IVP $y'' + 5y' + 4y = 3 - 2x$, $y(0) = 2, y'(0) = 1$.

The written exam takes place on **Saturday, 9 July 2005, from 9:00 to 11:00, in the Little Lecture Hall** (Kleiner Hörsaal), Linnéstr. 5.