Mathematical Institute
University Leipzig
Summer term 2005

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ODE for Physicists - Homework 10

Due: June 21, 2005

30. (12 pts.) Using Laplace transform methods, find the solution $y = y(t)$ to the following initial-value problems:

    (a) $y'' - 3y' + 2y = \mathrm{e}^{3t}, \qquad y(0) = 1, y'(0) = 0.$

    (b) $ty'' + 2y' + ty = 0, \qquad y(0) = 1,\ y(\pi) = 0.$

    (c) $ty'' + (t-1)y' - y = 0, \qquad y(0) = 5,\ y(+\infty) = 0.$

    (d) $y'' - ty' + y = 1, \qquad y(0) = 1, y'(0) = 2.$

31. (4 pts.) Let $\omega, k \in \mathbb{R}$ with $k \neq \omega$. Use the Laplace transform to show that the general solution $y = y(t)$ to the initial-value problem $y'' + k^2 y = A\cos\omega t$, $y(0) = \alpha$, $y'(0) = \beta$, is given by

$$y(t) = \frac{A(\cos\omega t - \cos kt)}{k^2 - \omega^2} + \alpha\cos kt + \frac{\beta}{k}\sin kt,$$

    where α, $\beta \in \mathbb{R}$ are free parameters.