Power control policy on the SINR graph

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Random Networks Seminar

14th July, 2015

Introduction

Let Φ a point process modelling the location of the nodes of a network. For any $t \in \mathbb{Z}_+$ let $\Phi_T(t) \subset \Phi$ be the set of nodes that are transmitting at time t and $\Phi_R(t) = \Phi \setminus \Phi_T(t)$ the ones that are receiving. We define the SINR from a node $x \in \Phi_T(t)$ to a node $y \in \Phi_R(t)$ as

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- γ the interference suppression constant, N the noise.

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- The expected value of the delay time of successfully transmitting one package from one node to another is finite.
- The average velocity in which the package travels around the net is not zero.

The model

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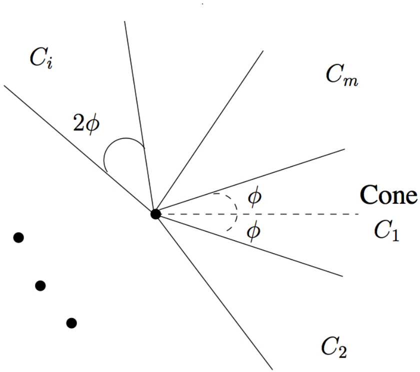
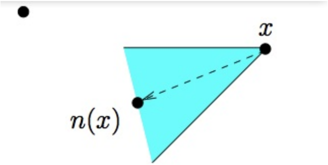
- $0 < \gamma < 1$.

We will track a tagged package that traverse the network following a conic forwarding strategy for that let $C_1, \ldots, C_m$ be cones centred in the origin with angle $2\Phi < \frac{\pi}{2}$ s.t. $\cup_{i=1}^{m} C_i = \mathbb{R}^2$ and are disjoint, also lets assume that C_1 is symmetric with respect to the x axis.



At time t the node x will transmit through the cone $x + C_d(x, t)$ that contains the final destination of the package to $n_t(x)$ the nearest node in that cone.



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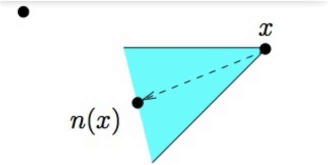

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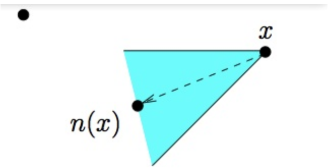

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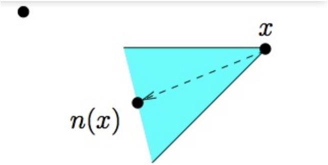

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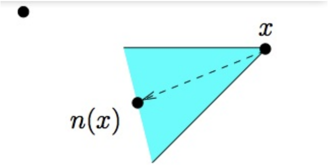

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Which implies that $p_x(t) = (1-\varepsilon)\ell(x, n_t(x))$.
This is what is called power control strategy.

Then we have that the SINR from node x to node y at time t is given by

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The next indicator function tell us if the transmission has been successful

$$e_{xy}(t) := \begin{cases} 1 & 1 \text{ if } SINR_{xy} > \beta \\ 0 & \text{otherwise.} \end{cases}$$

Definition

Let the minimum exit time taken by any packet to be successfully transmitted from node x to its nearest neighbour $n(x)$ in the destination cone of the packet be:

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Theorem

Suppose $\beta\gamma < 1$ then the SINR graph with power control policy satisfies that $E(T(x)) < \infty$ for any $x \in \Phi$.

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$$P(T(o) > k \mid \Phi) = E\{\prod_{t=1}^{k} P(A(t) \cup B(t) \mid \mathcal{G}_k)\mathbf{1}_F|\Phi\}.$$

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- $A(t) := \{o \in \Phi_R(t)\}$.
- $B(t) := \{o \in \Phi_T(t), n(o) \in \Phi_R(t), SINR_{o,n(o)}(t) \leq \beta\}$.

On the event F since $P_o(t)\ell(o, n(o)) = c$ and $h_t(o, n(o)) \sim exp(\mu)$.

$$P(A(t)|\mathcal{G}_k) = 1 - p_o(1),$$

$$P(B(t) \mid \mathcal{G}_k) = p_o(1)q_{n(o)}(t)(1 - E\{e^{-\frac{\mu\beta}{c}(N+\gamma I(t))} \mid \mathcal{G}_k\}).$$

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In consequence since $p_{n(o)}(t) = \ell(n(o))(1-\varepsilon) \leq 1 - \varepsilon$

$$\begin{aligned} P(A(t) \cup B(t)|\mathcal{G}_k) &\leq P(A(t)|\mathcal{G}_k) + P(B(t)|\mathcal{G}_k) \\ &\leq 1 - p_o(1)\varepsilon e^{-\frac{\mu\beta N}{c}} E(e^{-\frac{\mu\beta\gamma}{c} I(t)}|\mathcal{G}_k). \qquad (1) \end{aligned}$$

Let $a = \frac{\mu\beta\gamma}{c}$. Also suppose $z \in \Phi \setminus \{o, n(o)\}$ transmits using cone $z + C_i$ at time t with transmission probability $p_z^{(i)}(t)$ and power $P_z^{(i)}(t)$.

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$$E\{e^{-\frac{\mu\beta\gamma}{c}I(t)}|\mathcal{G}_k\} = \prod_{z\in\Phi\setminus\{o,n(o)\}} E\{e^{-a\mathbf{1}_z P_z^{(i)}(t)h_t(z,n(o))\ell(z,n(o))}|\mathcal{G}_k\}.$$

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Again from the fact that $h_t(z, n(o)) \sim exp(\mu)$

$$\begin{aligned} &E\{e^{-a\mathbf{1}_z P_z^{(i)}(t)h_t(z,n(o))\ell(z,n(o))}|\mathcal{G}_k\} \\ &= (1 - p_z^{(i)}) + p_z^{(i)} E\{e^{-aP_z^{(i)}(t)h_t(z,n(o))\ell(z,n(o))}|\Phi\} \\ &= (1 - p_z^{(i)}) + p_z^{(i)} \frac{c}{c + \beta\gamma\ell(z, n(o))P_z^{(i)}(t)}. \end{aligned} \tag{2}$$

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Since on the event F, $I^*(t) \triangleq I^*(1)$ then

$$P(A(t) \cup B(t) | \mathcal{G}_k) \leq 1 - p_o(1) \varepsilon e^{-\frac{\mu \beta N}{c}} E\{e^{-aI^*(1)} | \Phi\}.$$

Then for $J := p_o(1)\varepsilon e^{-\frac{\mu\beta N}{c}} E\{e^{-aI^*(1)}|\Phi\}$ we get that

$$P(T(o) > k|\Phi) \leq (1-J)^k$$

and since $0 < 1 - J < 1$

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$$\begin{aligned} E(T(o)) &= \sum_{k\geq 0} P(T(o) > k) \\ &= E(\sum_{k\geq 0} P(T(o) > k|\Phi)) \leq E(J^{-1}). \end{aligned}$$

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By Cauchy-Schwartz on J^{-1}

$$E(T(o)) \leq \frac{e^{\frac{\mu\beta N}{c}}}{\varepsilon}(E\{p_o(1)^{-2}\}E\{\frac{1}{(E\{e^{-aI^*(1)}|\Phi\})^2}\})^{\frac{1}{2}}.$$

From the definition of the transmission probability $p_o(t)$,

$$E\{p_o(1)^{-2}\} = E\{(\frac{c}{M})^2(|n(o)|^{2\alpha} \vee 1)\} < \infty,$$

since the nearest neighbour distance in a cone of an homogeneous PPP with intensity λ has density

$$f(r) = \frac{2\lambda\pi r}{m} e^{-\frac{\lambda\pi}{m} r^2}, r > 0.$$

For finishing the proof we have to show that

$$E\{\frac{1}{(E\{e^{-al^*(1)}|\Phi\})^2}\} < \infty,$$

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$$E\{e^{-a\mathbf{1}_z^* P_z^* h_1(z,n(o))\ell(z,n(o))}|\Phi\} = 1 - \frac{\beta\gamma p_z^* P_z^* \ell(z,n(o))}{c + \beta\gamma P_z^* \ell(z,n(o))}$$
$$\geq 1 - \frac{\beta\gamma M \ell(z,n(o))}{c} = 1 - \beta\gamma(1-\varepsilon)\ell(z,n(o)). \qquad (3)$$

Since $p_z^* P_z^* = M$ and $c = M(1-\varepsilon)^{-1}$.

Let $c_1 = \beta\gamma(1-\varepsilon)$ by (3) we get

$$\frac{1}{(E\{e^{-aI^*(1)}|\Phi\})^2} \leq \prod_{z\in\Phi\setminus\{o,n(o)\}} \frac{1}{(1-c_1\ell(z,n(o)))^2}. \qquad (4)$$

(*) Let Φ_0 a PPP independent of the other nodes and intensity $\lambda\mathbf{1}_{\{(o+C_d)\cap B(o,|n(o)|\}}$ i.e. $(\Phi\setminus\{o,n(o)\})\cup\Phi_0$ is a PPP of intensity λ with the origin at $n(o)$.

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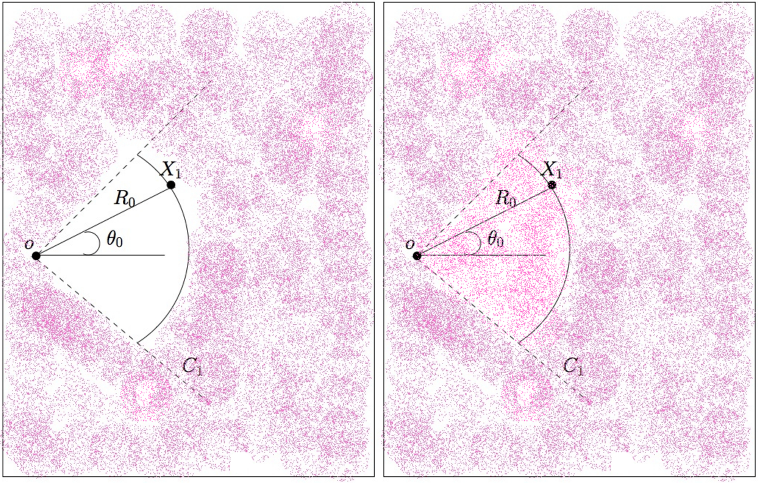
Since for $z\in\mathbb{R}^2$, $e^{-2log(1-c_1\ell(|z|)} \geq 1$ and by 4 we get

$$E\{\frac{1}{(E\{e^{-al^*(1)}|\Phi\})^2}\} \leq E\{\prod_{z\in\Phi\setminus\{o,n(o)\}\cup\Phi_0} e^{-2\log(1-c_1\ell(|z|))}\}.$$


X_1
R_0
θ_0
o
C_1
X_1
R_0
θ_0
o
C_1

Finally by Campbell´s theorem since $\int_{\mathbb{R}^2} \ell(|z|)dz < \infty$:

$$E\{\frac{1}{(E\{e^{-al^*(1)}|\Phi\})^2}\} \leq \exp(\lambda \int_{\mathbb{R}^2} (e^{-2log(1-c_1\ell(|z|))} - 1))dz)$$
$$\leq \exp(\frac{2\lambda c_1}{(1-c_1)^2} \int_{\mathbb{R}^2} \ell(|z|)dz) < \infty$$

Now we want to measure how fast the package moves in time from the origin to its destination.

Definition

Let T_0 be the time taken by this tagged package starting at $X_0 = o \in \Phi$ to successfully reach its nearest neighbour $X_1 = n(o)$ in the destination cone C_1. More generally let T_{i-1} be the time taken for the packet to successfully reach the nearest neighbour X_i of X_{i-1} in the destination cone $X_{i-1} + C_1$.

Now we want to measure how fast the package moves in time from the origin to its destination.

Definition

Let T_0 be the time taken by this tagged package starting at $X_0 = o \in \Phi$ to successfully reach its nearest neighbour $X_1 = n(o)$ in the destination cone C_1. More generally let T_{i-1} be the time taken for the packet to successfully reach the nearest neighbour X_i of X_{i-1} in the destination cone $X_{i-1} + C_1$.

Definition

The information velocity of SINR network is defined as

$$v = \liminf_{t \to \infty} \frac{d(t)}{t}$$

where $d(t)$ is the distance of the tagged packet from the origin at time t.

Theorem

Under the conditions of Theorem 2. the information velocity $v > 0$ a.s.

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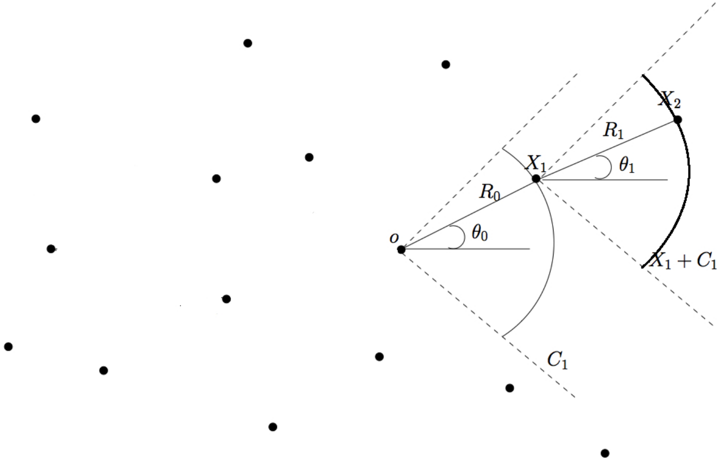
The cones $\{(X_i + C_1) \cap B(X_i, R_i), i \geq 0\}$ are non-overlapping since $2\phi < \frac{\pi}{2}$.


X_2
R_1
X_1
θ_1
R_0
θ_0
o
$X_1 + C_1$
C_1

Since Φ is homogeneous PPP with intensity λ, we have that $\{(R_i, \theta_i), i \geq 0\}$ is an i.i.d. sequence of random vectors where

- R_i has density
$$f(r) = \frac{2\lambda\pi r}{m} e^{-\frac{-\lambda\pi}{m} r^2},$$

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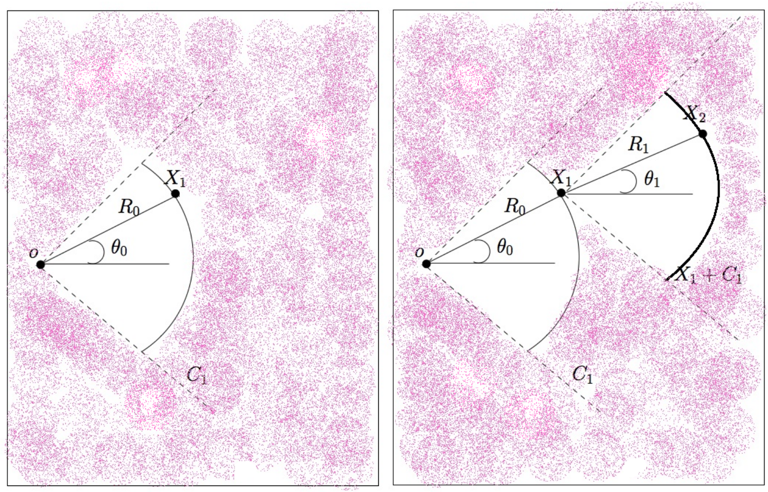
(*)Our goal is to construct an stationary sequence of stopping times such that for all $i \geq 0$, $T_i^{'} \geq T_i$.



o
R_0
X_1
θ_0
C_1
o
R_0
θ_0
X_1
R_1
θ_1
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C_1

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- $\{(R_{-i}, \theta_{-i}, i \geq 1)\}$ an i.i.d sequence of random vectors with distribution (R_0, θ_0).

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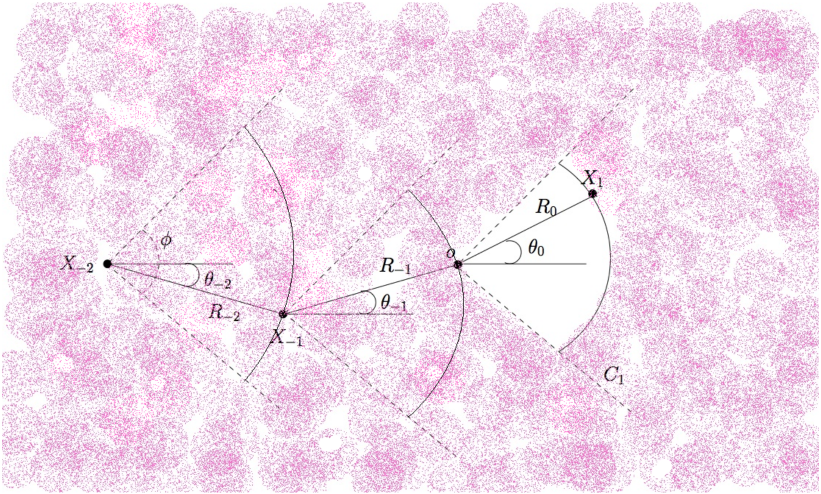
X_{-2}
ϕ
θ_{-2}
R_{-2}
X_{-1}
θ_{-1}
R_{-1}
o
R_0
θ_0
X_1
C_1

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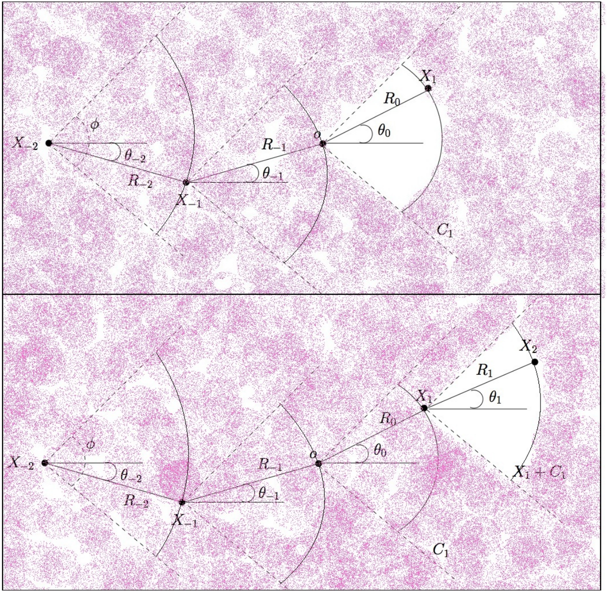
$(T_i^{'}, i \geq 0)$ is a stationary sequence with $T_i^{'} \geq T_i$. We want to proof that $E(T_0^{'}) < \infty$ and then use the Birkoff´s ergodic theorem.


X_1
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C_1
X_2
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R_0
θ_0
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C_1

Let $\widetilde{I}(t) = \sum_{z \in \widetilde{\Phi}} \mathbf{1}_z P_z(t) h_t(z, n(o)) \ell(z, n(o))$. Analogously to the first theorem we have that

$$E(T_0^{'}) \leq \frac{e^{\frac{\mu \beta N}{c}}}{\varepsilon} (E\{p_o(1)^{-2}\} E\{\frac{1}{(E\{e^{-a(I^*(1)+\widetilde{I}^*(1))} | \Phi \cup \widetilde{\Phi}\})^2}\})^{\frac{1}{2}}.$$

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Since we conditioned on $\Phi \cup \widetilde{\Phi}$, $I^*(1)$ and $\widetilde{I}^*(1)$ are independent we have that

$$E\{e^{-a(I^*(1)+\widetilde{I}^*(1))} | \Phi \cup \widetilde{\Phi}\} = E\{e^{-aI^*(1))} | \Phi\} E\{e^{-a\widetilde{I}^*(1)} | \widetilde{\Phi} \cup \{n(o)\}\}$$

By Cauchy-Schwartz inequality the result follows if we show

$$E\{\frac{1}{(E\{e^{-aI^*(1))}|\Phi\})^4}\}E\{\frac{1}{(E\{e^{-a\widetilde{I}^*(1)}|\widetilde{\Phi}\cup\{n(o)\}\})^4}\}<\infty.$$

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Again by Campbell's theorem

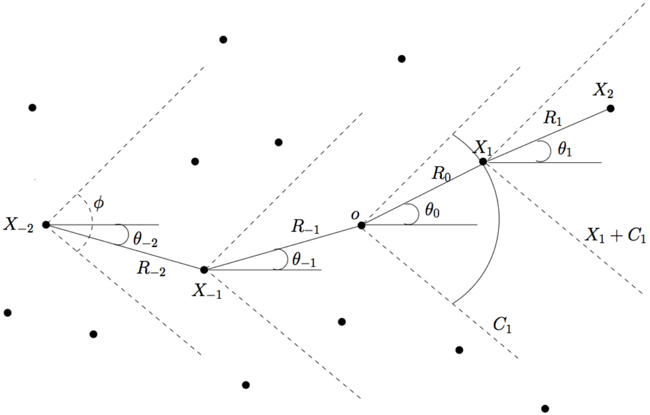
$$E\{\frac{1}{(E\{e^{-aI^*(1))}|\Phi\})^4}\} \leq \exp(\frac{\lambda}{(1-c_1)^4}\int_{\mathbb{R}^2}(1-(1-c_1\ell(|z|))^4)dz) < \infty$$

.

(*)Since for all $i \in \mathbb{N}$, $\sum_{j=0}^{i} R_{-j}\cos(\theta_{-j}) \leq |X_{-i} - n(o)|$ and ℓ is decreasing

$$E\{\frac{1}{(E\{e^{-a\widetilde{I}^*(1)}|\widetilde{\Phi} \cup \{n(o)\}\})^4}\} \leq E\{\prod_{i=1}^{\infty} e^{-4\log(1-c_1\ell(X_{-i},n(o)))}\}$$
$$\leq E\{\prod_{i=1}^{\infty} e^{-4\log(1-c_1\ell(\sum_{j=0}^{i} R_{-j}\cos(\theta_{-j}))}\}$$
$$= E\{e^{\sum_{n=1}^{\infty} g(S_{n+1})}\}$$

where $S_n = \sum_{j=0}^{n-1} R_{-j}\cos(\theta_{-j})$ and $g(x) = -4\log(1 - c_1\ell(x))$.


X_2
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$X_1 + C_1$
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Let $0 < \delta < E\{R\cos(\theta)\}$, by the Chernoff bound

$$P(\frac{S_n}{n} < \delta) \leq e^{-\zeta(\delta)n},$$

with $\zeta(\delta) = \sup_{v \leq 0}\{\nu\delta - \log(E(e^{\nu R\cos(\theta)}))\}$.

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Then by the Borel Cantelli lemma exists $N(\omega)$, and $c_2 > 0$ s.t.

$$P(N \geq m) = P(S_n < n\delta \text{ for some } n \geq m)$$
$$\leq \sum_{n=m}^{\infty} e^{-\zeta(\delta)n} \leq c_2 e^{-\zeta(\delta)m}.$$

Since g is non-increasing, we get

$$E\{e^{\sum_{n=1}^{\infty} g(S_n)}\} = E\{e^{\sum_{n=1}^{N} g(S_n)+\sum_{n=N+1}^{\infty} g(S_n)}\}$$
$$\leq E\{e^{\sum_{n=1}^{N} g(0)+\sum_{n=N+1}^{\infty} g(n\delta)}\} \leq e^{\sum_{n=1}^{\infty} g(n\delta)} E\{e^{g(0)N}\}.$$

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- On the one hand by the comparison test $\sum_{n=1}^{\infty} g(n\delta) < \infty$.
- On the other hand since $R\cos(\theta) > 0$ then $\zeta(\delta) \uparrow \infty$ as $\delta \downarrow 0$, and we can choose δ s.t. $\zeta(\delta) > g(0)$ so it follows that $E\{e^{g(0)N}\} < \infty$.

By Birkoffs ergodic theorem exist a r.v. T' s.t.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=0}^{n-1} T'_k = T'.$$

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Also since $T_i^{'} \geq 1$ then $T^{'} \geq 1$.
Finally from the fact that $\mathbf{1}_{T^{n-1} \leq t < T^n} d(t) \geq \sum_{k=1}^{n-1} R_k \cos(\theta_k)$, we conclude that

$$\liminf_{t\to\infty} \frac{d(t)}{t} \geq \lim_{n\to\infty} \frac{\sum_{k=1}^{n-1} R_k \cos(\theta_k)}{\sum_{k=1}^{n-1} T_k^{'}} = \frac{E(R\cos(\theta))}{T^{'}} > 0,$$

as we wanted.

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