Berlin Leipzig Seminar
Analysis/probability theory
Fourth Meeting Winter Term 2004/05

Jointly organized by the University of Leipzig, the Max Planck Institute Leipzig, and the Technical University Berlin

Date:

Friday, 4 February 2005

Venue:

Technical University Berlin

Institute for Mathematics

Str. des 17. Juni 136

10623 Berlin

Room MA313/314

Program:

10:20–11:10: **Maria Reznikoff (University Bonn)**

Stochastically driven phase transformation: the effect of time constraints and domain size on switching pathways

11:10–12:00: **Nicolas Dirr (Max Planck Institute Leipzig)**

Instability of a front under forced displacements

12:00–12:50: **Max-Konstantin von Renesse (Technical University Berlin)**

On Wasserstein geometry and SPDE on measure space

Everybody is welcome to attend.

Wolfgang König, University of Leipzig

Abstracts:

Reznikoff:

Phase transformation which would never occur in the deterministic setting becomes possible in the presence of a stochastic perturbation. For instance, the stable state $u \equiv -1$ of the deterministic Allen-Cahn equation can be stochastically driven to the opposing state, $u \equiv +1$. In the first part of the talk, we consider the Allen-Cahn phase transformation problem and the related large deviation action functional,

$$\int_0^T \int_{[0,L]^d} |\dot{u} + (-\Delta u + V'(u))|^2 \, dx \, dt.$$

We discuss the scaling of the minimum action as a function of the length- and time-scales of the system and explore the sharp-interface limit of the functional in the critical scaling, $L \sim \sqrt{T}$. In the second part, we consider the very different problem of phase transformation on large or infinite domains. Here, we study the one-dimensional equation,

$$\dot{u} \, = \, u_{xx} - V'(u) + \sqrt{2\varepsilon}\, \eta,$$

where V is an *unequal*-well potential and η is a space-time white noise. We use partially heuristic arguments to reduce the transformation problem (in a suitable limit) to a simple form of the JMAK nucleation and growth model.

Dirr:

By applying linear response theory and the Onsager principle, the power (per unit area) needed to make a planar interface move with velocity V is found to be equal to V^2/μ, μ a mobility coefficient. To verify such a law, we study a one dimensional model where the interface is the stationary solution of a non local evolution equation, called an instanton. We then assign a penalty functional to orbits which deviate from solutions of the evolution equation and study the optimal way to displace the instanton. We find that the minimal penalty has the expression V^2/μ only when V is small enough. Past a critical speed, there appear nucleations of the other phase ahead of the front, their number and location are identified in terms of the imposed speed. We explain the connections to large deviations and to Gamma-convergence.

(Joint work with A. De Masi and E. Presutti)

von Renesse:

We review some recent results on the geometry of the space of measures on a Riemannian manifold and its connection to optimal mass transportation. We also give an outlook to the construction of diffusion processes on the space of measures and the connection to conservative stochastic PDE, such as stochastic heat or stochastic Cahn-Hilliard equation.