Berlin Leipzig Seminar
Analysis/probability theory
First Meeting Winter Term 2005/06

Jointly organized by the University of Leipzig, the Max-Planck Institute Leipzig, and the Technical University Berlin

Date:

Friday, 28 October 2005

Venue:

Technical University Berlin, Institute for Mathematics, Str. des 17. Juni 136, 10623 Berlin
Room MA313

Program:

9:00–10:00: **Steffen Dereich (Technical University Berlin)**
High resolution coding of stochastic processes

10:10–11:10: **Elliott Lieb (Princeton University)**
The Bose gas

11:20–12:20: **Wolfgang König (University of Leipzig)**
Many-particle limit for symmetrised traces

Everybody is welcome to attend.

Wolfgang König, University of Leipzig

Abstracts:

Dereich: *High resolution coding of stochastic processes*

Let X denote a stochastic process (the *original*) and n a natural number. We consider the finding of a good approximation $\hat{X}$ to X (*reconstruction*) that attains at most n different elements (the *quantization problem*). As measure for the discrepancy between the original and the reconstruction (*distortion*) we consider a finite moment of a Banach space distance $\|X-\hat{X}\|$.

In the first part of the talk, we summarize recent results for the asymptotic quantization problem (the *high-resolution quantization problem*) for Gaussian processes. We analyze optimal quantization strategies and compare the results with a particular random strategy in quantization.

In the second part we will treat the quantization problem for solutions X of a stochastic differential equation. As distortion we consider again an arbitrary moment of the $L^p[0,1]$- and supremum-norm distance. We shall see that the problem is closely related to the quantization problem for Brownian motion. Based on this link we derive the strong asymptotics of the best-achievable distortion when n tends to infinity.

Lieb: *The Bose gas*

The peculiar quantum-mechanical properties of the ground state of quantum-mechanical many-body systems (bosons) at low density and temperature can now be examined experimentally; this continues to be an active and exciting topic. The mathematical derivation of these results is difficult because the problem is extremely quantum mechanical – meaning that the mathematical subtleties of the Schroedinger equation are richly displayed. A survey, including recent results (with Seiringer, Solovej and Yngvason) will be given. One of the interesting open problems is the question of Bose-Einstein condensation and its relation to Wiener integrals.

König: *Many-particle limit for symmetrised traces*

The Hamiltonian $\mathcal{H}_N = \sum_{i=1}^N(-\Delta_{x_i} + W(x_i))$ describes the kinetic energy of N particles in a trap $W\colon \mathbb{R}^d \to [0,\infty]$ satisfying $\lim_{|x|\to\infty} W(x) = \infty$. We are interested in the large-N asymptotics of the symmetrized trace of the operator $\mathrm{e}^{-\beta\mathcal{H}_N}$ for fixed $\beta \in (0,\infty)$, i.e., the trace of the projection of that operator to the set of symmetric wave functions. This projection describes N Bosons in the trap W at positive temperature $1/\beta$. It is well-known that this trace may be represented in terms of N Brownian bridges $[0,\beta] \to \mathbb{R}^d$ with uniformly permuted ending and starting locations. We give precise large-N exponential rates of the empirical path measure of the N paths and of the mean of their occupation measures, in terms of a large-deviation principle. In the latter case, the rate function turns out to admit an explicit formula involving the principal eigenvalue of $-\Delta + W$.

This is joint work in progress with Stefan Adams (MPI Leipzig).