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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 10 (last problems)

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. *Gradient of a linear function on a triangle.* Consider a linear function u^h on the triangle K with the vertices $P_i = (x_i, y_i), i = 1, 2, 3$, which has the values $u^h(P_i)$. Find a formula for ∇u^h.

2. *Norm for a non-conforming finite element space.* Let P_1^{nc} the two-dimensional Crouzeix–Raviart finite element space with functions that vanish in the midpoints of the edges which lie on the boundary of the domain. Prove that
$$\|v_h\|_h = \left(\sum_{K \in \mathcal{T}_h} \|\nabla v_h\|^2_{L^2(K)} \right)^{1/2}$$
defines a norm in this space.
Hint: It is clear that $\|v_h\|_h$ defines a seminorm. One has to show that from $\|v_h\|_h = 0$, it follows that $v_h = 0$.

3. *Properties of the interpolation operator.* Show that the interpolation operator $I_{\hat{K}} \ : \ C^s(\hat{K}) \to \hat{P}(\hat{K})$, which was defined in the lecture, is a linear and continuous operator. In addition, show that the restriction of $I_{\hat{K}}$ to $P(\hat{K})$ is the identity operator.
This problem can be solved only after the class on July 01, 2019.

Remind the programming problem from Exercise Problems 09!

The exercise problems should be solved in groups of two or three students. The written parts have to be submitted until **Thursday, July 04, 2019**. The executable codes have to be send by email to A. Jha.