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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 02

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. *Classification of second order partial differential equations.* Classify the following partial differential equations

$$\begin{aligned}
&\text{i)} && \partial_{xx}u + 2\partial_{xy}u + 2\partial_{yy}u + 4\partial_{yz}u + 5\partial_{zz}u + \partial_x u + \partial_y u &&= 0, && (3d)\,,\\
&\text{ii)} && e^z\partial_{xy}u - \partial_{xx}u - \log\left(x^2+y^2+z^2\right) &&= 0, && (3d)\,,\\
&\text{iii)} && \partial_{xx}u + 4\partial_{xy}u + 3\partial_{yy}u + 3\partial_x u - \partial_y u + 2u &&= 0, && (2d)\,,\\
&\text{iv)} && a\partial_{xx}u + 2a\partial_{xy}u + a\partial_{yy}u + b\partial_x u + c\partial_y u + u &&= 0, && (2d)\,,
\end{aligned}$$

   $2d$ – in two dimensions, $3d$ – in three dimensions.

2. *Basic properties of finite difference approximations.* Solve the following problems.

   i) Show that
$$v_{\hat{x},i} = \frac{1}{2}\left(v_{x,i} + v_{\bar{x},i}\right), \quad v_{\bar{x}x,i} = \left(v_{\bar{x},i}\right)_{x,i}.$$

   ii) Consider a function $v(x)$ at x_i and show the following consistency estimates
$$v_{\hat{x},i} = v'(x_i) + \mathcal{O}\left(h^2\right), \quad v_{\bar{x}x,i} = v''(x_i) + \mathcal{O}\left(h^2\right).$$

   iii) Compute the order of consistency of the following finite difference approximation

$$u''(x) \sim \frac{1}{12h^2}\Big(-u(x+2h) + 16u(x+h) - 30u(x) + 16u(x-h) - u(x-2h)\Big).$$

The exercise problems should be solved in groups of two or three students. The written parts have to be submitted until **Thursday, Apr. 25, 2019** to A. Jha. The executable codes have to be send by email to A. Jha.