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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 04

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Show that the vector $v_k = (v_{k,0}, \cdots, v_{k,n})$ with

$$v_{k,0} = v_{k,n} = 0, \quad v_{k,i} = \sqrt{2}\,\sin\left(\pi\,k\,x_i\right),$$

solves the eigenvalue problem

$$v_{k,i-1} + \left(\lambda_k\,h^2 - 2\right) v_{k,i} + v_{k,i+1} = 0$$

with

$$\lambda_k = \frac{2}{h^2}\left(1 - \cos\left(\pi\,k\,h\right)\right) = \frac{4}{h^2}\sin^2\left(\frac{\pi\,k\,h}{2}\right).$$

2. The lecture notes contain a revised and more detailed proof of Theorem 2.43 (convergence of the higher order finite difference scheme) than given in the class. In this proof, the following argument is used: Let $A, B \in \mathbb{R}^{n\times n}$ be two symmetric and positive definite matrices with $AB = BA$ and

$$(A\mathbf{x}, \mathbf{x}) \geq (B\mathbf{x}, \mathbf{x}) \quad \forall\ \mathbf{x} \in \mathbb{R}^n,$$

then $\|A\mathbf{x}\|_2 \geq \|B\mathbf{x}\|_2$ for all $\mathbf{x} \in \mathbb{R}^n$. The symbols denote the Euclidean inner product and the Euclidean vector norm. Prove this statement.

3. **This problem has to be solved until May 14, 2013.** The problem is formulated analogously as Problem 3 from Exercise Problems 03. The only difference is that the finite difference scheme with the nine point stencil should be used (instead of using the five point stencil).

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Tuesday, May 07, 2013** either before or after one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.