Exercises to the classes
Numerical Methods in Sciences and Technics
Exercises no. 11
to 19.01.2004

The solution of exercise 2 is to submit in the exercise classes on Monday, 19.01.2004!

Statements given in the lecture can be used in the solution of the exercises without proof. All other statements have to be proved.

1. Prove Gronwall’s lemma: Let $\phi(t) : [t_0, t_e] \to \mathbb{R}$ be continuous with

$$\phi(t) \leq \alpha + \beta \int_{t_0}^{t} \phi(\xi) \mathrm{d}\xi$$

for all $t \in [t_0, t_e]$, $\alpha, \beta \in \mathbb{R}$, $\alpha, \beta > 0$. Then

$$\phi(t) \leq \alpha \exp(\beta(t - t_0)).$$

Hint: use the result of

$$\frac{d}{dt} \ln \left( \alpha + \beta \int_{t_0}^{t} \phi(\xi) \mathrm{d}\xi \right).$$

2. Compute the differentiation index of the DAE

$$0 = g(v)$$
$$v' = f(v, w)$$
$$w' = k(v, w, u)$$

where $g, f, k$ has a bounded inverse.