Faculty of Mathematics
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Exercises to the classes
Numerical Methods in Sciences and Technics

Exercises no. 10

to 12.01.2004

The solution of exercise 2 is to submit in the exercise classes on Monday, 12.01.2004 !

Statements given in the lecture can be used in the solution of the exercises without proof. All other statements have to be proved.

1. Show that for each positive definite symmetric matrix $A \in \mathbb{R}^{n\times n}$ holds

$$\|e\|_1 = \frac{1}{2}\sum_{i,j=1}^{n}(-a_{ij})(e_i - e_j)^2 + \sum_{i=1}^{n} s_i e_i^2$$

   with

$$s_i = \sum_{j=1}^{n} a_{ij}.$$

2. Consider a finite difference discretisation of the Laplace operator on the unit square such that the nodes are ordered as a two-dimensional array (see the example in Section 1.7.5) and for the inner nodes is

$$a_{ij} = \begin{cases} 8 & \text{if } i = j \\ -1 & \text{if } j \text{ is a neighbour of } i \\ 0 & \text{else.} \end{cases}$$

   A node v_j is a neighbour of v_i if v_j is either on top, on the bottom, on the left, on the right of v_i or it is in one of the four diagonal directions next to v_i.

   Let $\varepsilon = 0.125$. Sketch the standard coarsening process on a 5×5 patch of nodes. The first C-node is the node in row 2 and in column 2. (Compare for the general approach the example in Section 1.7.5)