Exercises to the classes
Numerical Methods in Sciences and Technics

Exercises no. 3
to 03.11.2003

The solution of exercise 3 is to submit in the exercise classes on Monday, 03.11.2003!

Statements given in the lecture can be used in the solution of the exercises without proof. All other statements have to be proved.

1. Let $S_\omega$ be the iteration matrix of the damped Jacobi iteration. Show that $S_\omega$ has the same eigenvectors as the matrix of the model problem.

2. Write a matlab script for the SOR iteration. Consider the model problem with $a = 0$ and $f = 0$ on a mesh with $N = 128$. Do 100 iterations with the damping factors $\omega = 1$ and $\omega = 1.9$ and the initial guess $u_0 = (u_0^1, \ldots, u_0^{N-1})^T$ with

$$u_j^0 = \sin \left( \frac{j k \pi}{N} \right), \quad j = 1, \ldots, N - 1$$

for $k \in \{1, 3, 10, 64\}$. Compute the error $\|e^{100}\|_\infty$.

3. Consider the $k$-th Fourier mode, $N/2 < k < N$, on a grid $\Omega^h$ with $N$ intervals. Show that this mode is represented on the grid $\Omega^{2h}$ (with $N/2$ intervals) as the negative of the $(N - k)$-th mode on $\Omega^{2h}$.