Fachbereich Mathematik und Informatik
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Numerical Methods for Convection-Dominated Problems

Exercise Problems 05

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Solve the following problems.

   (a) Let the condition of Remark 5.8 be satisfied and let $\boldsymbol{b}, \nabla \cdot \boldsymbol{b}, c \in L^\infty(\Omega)$. Show estimate (5.8), where the constant M does not depend on negative powers of ε.

   (b) Consider Example 5.10. There, the equivalence of the Petrov–Galerkin finite element scheme and the finite difference scheme is stated. Show that the matrix entries $a_{i-1,i}$ are the same of both schemes, apart of a multiplicative factor h.

   8 points

2. Download the code `model_problem_2d` and the text files `model_problem_2d_readin.dat`, `UnitSquare.PRM` from the homepage of the course.

   The code solves the convection-diffusion problem

$$\begin{aligned} -\varepsilon \Delta u + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cdot \nabla u &= 1 \quad \text{in } \Omega = (0,1)^2, \\ u &= 0 \quad \text{on } \partial\Omega. \end{aligned}$$

   The parameters in `model_problem_2d_readin.dat` are set such that $\varepsilon = 10^{-6}$ and Q_1 finite elements are used on a grid consisting of squares.

   The SUPG method is applied.

   - Execute
     `chmod a+x model_problem_2d`
     and then run the code
     `./model_problem_2d model_problem_2d_readin.dat`.
   - Compute the solution for different values of the factor in the SUPG parameter $C_0 \in \{0, 0.1, 0.25, 0.5, 1, 2, 10, 100\}$. The value $C_0 = 0$ corresponds to the Galerkin finite element method. The value of C_0 can be set in `model_problem_2d_readin.dat`.
   - The minimal and maximal value of the computed solution on each level are given, see `extremal values`. The output of the maximal value gives the difference of this value and 1. The maximal value of the solution of the continuous problem is slightly less than 1.

- Data for graphical outputs are written. They can be visualize either with `paraview` or `gnuplot`.

Based on the pictures, describe the computed solutions with respect to the choice of C_0. What changes if C_0 is changed? Which value is in your opinion the best value for C_0? **8 Punkte**

The exercise problems should be solved in groups of two or three students. They have to be submitted until **July 11, 2016** either by email or in the morning lecture.