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Numerical Methods for Convection-Dominated Problems

Exercise Problems 04

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Extend the code written for Problem 3.2 to

   - the Iljin–Allen–Southwell scheme

   and perform the same investigations as in Problem 3.2. **4 points**

2. Extend the code for the simple upwind scheme to a Shishkin mesh.

   - The problems to be solved are the same as in Problem 3.2.
   - The transistion points are defined by $\sigma \in \{0.5, 1, 2, 4\}$.

   Hint: It is simpler to program the simple upwind scheme in the form with the backward finite difference. **8 points**

3. Consider the space $L^2(0,1)$, the bilinear form

$$a \ : \ L^2(0,1) \times L^2(0,1) \to \mathbb{R}, \quad a(u,v) = \int_0^1 x u(x) v(x) \ dx,$$

   and the linear functional

$$f \ : \ L^2(0,1) \to \mathbb{R}, \quad f(v) = \int_0^1 v(x) \ dx.$$

   a) Show that a and f are bounded.
   b) Does $a(\cdot,\cdot)$ satisfy all assumptions of the Theorem of Lax–Milgram ? Support your statement with an example.

   4 Punkte

The exercise problems should be solved in groups of two or three students. They have to be submitted until **July 04, 2016** either by email or in the morning lecture.