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Numerical Methods for Convection-Dominated Problems

Exercise Problems 03

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Solve the following problems.
   (a) Show that the discrete solution of Example 3.30 possesses the given form.
   (b) Show that the discrete solution of Example 3.40 possesses the given form.
   (c) Show that the functions

$$\sigma(q) = \sqrt{1+q^2}, \quad \sigma(q) = 1 + \frac{q^2}{1+q}.$$

   satisfy the assumptions of Theorem 3.47 (lecture on June, 06).

2. Write a code, e.g., in MATLAB, that approximates the solution of the two-point boundary value problem from Example 2.8 (the standard example).
   - Use as diffusion coefficient $\varepsilon \in \{1, 10^{-3}, 10^{-6}, 10^{-9}\}$.
   - Apply the following methods:
     - the central difference scheme,
     - the simple upwind scheme,
     - the Samarskij upwind scheme (lecture on June, 06).
   - The simulations should be performed on grids with

$$N \in \{4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048\}$$

     intervals. Compute the error to the solution of the continuous problem in the discrete maximum norm. Based on the error, compute the numerical order of convergence k.
     A formula for k can be obtained from the ansastz

$$\|u - u_h\|_{\infty,d} = ch^k,$$

     by solving this ansatz for k for the mesh widths h and $2h$ (with the same constant c).

12 points

The exercise problems should be solved in groups of two or three students. They have to be submitted until **June 13, 2016** either by email or in the morning lecture.