Fachbereich Mathematik und Informatik
Freie Universität Berlin
Prof. Dr. V. John
john@wias-berlin.de

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Numerical Methods for Convection-Dominated Problems

Exercise Problems 02

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Show the orders of consistency for the finite difference operators given in Example 3.7. **4 points**

2. Consider the differential operator

$$Lu = \frac{d}{dx}\left(k(x)\frac{du}{dx}\right),$$

where $k(x)$ is two times continuously differentiable. Define the finite difference operator L_h as follows

$$(L_h u_h)_i \quad := \quad D^+(aD^- u(x_i)),$$

where a is a grid function that should be chosen appropriately.

Show that the choices

$$a_i = \frac{k_i + k_{i-1}}{2}, \quad a_i = k\left(x_i - \frac{h}{2}\right),$$

define finite difference operators that are consistent of second order. Show also that the natural choice $a_i = k_i$ gives a a finite difference operator that is only of first order consistent. **8 points**

3. Let $A = (a_{ij}) \in \mathbb{R}^{n \times n}$ be an M-matrix. Show that $a_{ii} > 0$, $i = 1, \ldots, n$. **4 points**

The exercise problems should be solved in groups of two or three students. They have to be submitted until **May 23, 2016** either by email or in the morning lecture.