1. Consider the problem
\[-\varepsilon u'' + b(x)u' + c(x)u = f(x), \quad \text{for } x \in (0, 1), \ u(0) = u(1) = 0.\]
Show that the transform
\[\tilde{u}(x) := u(x) \exp \left( -\frac{1}{2\varepsilon} \int_0^x b(\xi) \, d\xi \right), \quad x \in [0, 1],\]
leads to a problem of symmetric form
\[-\varepsilon \tilde{u}''(x) + \tilde{c}(x)\tilde{u}(x) = \tilde{f}(x), \quad x \in (0, 1), \ \tilde{u}(0) = \tilde{u}(1) = 0\]
with the functions \(\tilde{c}(x), \tilde{f}(x)\) given in the lecture notes, Remark 2.9. 4 points

2. Compute the solution of
\[-\varepsilon u'' - u' = 1 \quad \text{in } (0, 1), \ u(0) = u(1) = 0, \ \varepsilon > 0\]
and the solution \(u_0\) of the corresponding reduced problem. 4 points

3. Compute the solution \(u(x)\) of the differential equation
\[-u'' + u' - u = 1,\]
with respect to the following intervals and boundary values:
\[
\begin{align*}
\Omega &= \left(0, \frac{2\pi}{3}\right) \quad u(0) = 0 \quad u \left(\frac{2\pi}{3}\right) = 0, \\
\Omega &= \left(0, \frac{2\pi}{3}\right) \quad u(0) = 1 \quad u \left(\frac{2\pi}{3}\right) = -2e^{\frac{2\pi}{3}} - 1, \\
\Omega &= (0, 1) \quad u(0) = 1 \quad u(1) = 1.
\end{align*}
\]
4 points

The exercise problems should be solved in groups of two or three students. They have to be submitted until May 02, 2016 either by email or in the morning lecture.