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Numerical Methods for Convection-Dominated Problems

Exercise Problems 01

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Consider the problem

$$-\varepsilon u'' + b(x)u' + c(x)u = f(x), \quad \text{for } x \in (0,1),\ u(0) = u(1) = 0.$$

Show that the transform

$$\tilde{u}(x) := u(x) \exp\left(-\frac{1}{2\varepsilon}\int_0^x b(\xi)\ d\xi\right), \quad x \in [0,1],$$

leads to a problem of symmetric form

$$-\varepsilon\tilde{u}''(x) + \tilde{c}(x)\tilde{u}(x) = \tilde{f}(x), \quad x \in (0,1), \tilde{u}(0) = \tilde{u}(1) = 0$$

with the functions $\tilde{c}(x), \tilde{f}(x)$ given in the lecture notes, Remark 2.9. **4 points**

2. Compute the solution of

$$-\varepsilon u'' - u' = 1 \quad \text{in } (0,1), \quad u(0) = u(1) = 0,\ \varepsilon > 0$$

and the solution u_0 of the corresponding reduced problem. **4 points**

3. Compute the solution $u(x)$ of the differential equation

$$-u'' + u' - u = 1,$$

with respect to the following intervals and boundary values:

$$\begin{array}{rclrclrcl} \Omega & = & \left(0, \frac{2\pi}{\sqrt{3}}\right) & u(0) & = & 0 & u\left(\frac{2\pi}{\sqrt{3}}\right) & = & 0, \\ \Omega & = & \left(0, \frac{2\pi}{\sqrt{3}}\right) & u(0) & = & 1 & u\left(\frac{2\pi}{\sqrt{3}}\right) & = & -2e^{\frac{\pi}{\sqrt{3}}} - 1, \\ \Omega & = & (0,1) & u(0) & = & 1 & u(1) & = & 1. \end{array}$$

4 points

The exercise problems should be solved in groups of two or three students. They have to be submitted until **May 02, 2016** either by email or in the morning lecture.