

Berlin, 12.01.2026

Numerik I

English translation of Übungsserie 10

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statements without reasoning won't get any points.

1. *Vector field.* Sketch the vector field for the ordinary differential equation $y'(x) = x^2 + y^2(x)$.

2 points

2. *Types of integrable first order ordinary differential equations.* Solve the following ordinary differential equations.

- a) $y'(x) + y^2(x) = 1,$
b) $y'(x) + y(x) \cos x = 0,$
c) $y'(x) = xy^7(x),$
d) $2y(x)y'(x) = x^2.$

4 points

3. *Solution by substitution.* Determine the general solution of

$$y'(x) = (x - y(x))^2 + 1.$$

Hint: Find a suitable substitution.

3 points

4. *Initial value problem with multiple solutions.* Demonstrate that solutions of the initial value problem

$$y'(x) = \sqrt{|y(x)|}, \quad y(0) = 0$$

are not unique.

4 points

The exercises should be solved in groups of two students. They have to be submitted until Sie **Wednesday, 21.01.2026, 10:00** electronically via whiteboard.