Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Numerik I

English translation of Übungsserie 06

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. *Spline interpolation* Consider the space of cubic splines with natural boundary conditions on the interval $[0, 2]$, which is decomposed into two equidistant subintervals. Which of the following functions is in the mentioned space?

   (a) $f(x) = x^3 - x^2$,

   (b) $f(x) = x^2(x-6) - (x-2)^3$,

   (c) $f(x) = (\max\{0, x-1\})^3 - 0.5x^3$.

   2 points

2. *Spline interpolation* Compute the cubic splice using natural boundary conditions and interpolation nodes

$$(0,0), \quad \left(\frac{\pi}{3}, \sin\left(\frac{\pi}{3}\right)\right), \quad \left(\frac{2\pi}{3}, \sin\left(\frac{2\pi}{3}\right)\right), \quad (\pi, 0),$$

   i.e. a cubic spline interpolation of the sin function. The calculations have to be done exactly i.e. not by the means of numerical approximation. It is sufficient to specify the spline in the form of (3.26) in the lecture notes. **6 points**

3. *Programming exercise.* Consider Runge's example (Beispiel 3.8).

   (a) Implement a program, which computes the interpolation polynomial for a equidistant grid with $n \in \{2, 10, 20\}$ intervals. Use the Newton basis for this purpose and depict the result graphically.

   (b) Implement a program, which computes the interpolation polynomial for a grid with $n \in \{2, 10, 50\}$ intervals. The nodes of the grid are supposed to be the roots of the Chebyshev polynomial of first kind. Depict the result graphically.

   (c) Approximate the error of all interpolation polynomials in the uniform norm. For this purpose subdivide $[-5, 5]$ in 10000 equidistant subintervals and choose the largest error at the grid nodes. Use Horner's method (appendix of the lecture notes, see the homepage of the lecture notes) for computing the values of the polynomial. What can you observe?

   6 points

The exercises should be solved in groups of two students. They have to be submitted until Sie **Wednesday, 03.12.2025, 10:00** electronically via whiteboard.