Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Numerik I

English translation of Übungsserie 02

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. *Different best approximations of a polynomial.* Let $V = C([-1,1])$, $f(x) = x^4$ and $U = P_3$ the space of polynomials of degree less or equal to 3 on $[-1,1]$. Compute the Chebyshev approximation as well as the best approximation in $L^2(-1,1)$ of f onto U. Determine the error of each approximation in both, the L^2-Norm and the maximum norm. **4 points**

2. *Properties of spaces and bases.* Solve the following problems.

   i) Let V be an inner product space with finite dimensional subspace $U \subseteq V$ and let $\{\varphi_i\}_{i=1}^n$ be a basis of U. Furthermore let $f \in V$ and $u \in U$. Prove that
   $$(f - u, v) = 0 \quad \forall\ v \in U$$
   is satisfied if and only if it is for every basis function of U.

   ii) Let $\{\varphi_i\}_{i=1}^n$ a Basis of U. Prove positivity and symmetry of the Gram matrix introduced in the lectures and given by
   $$A = (a_{ij})_{i,j=1}^n, \quad a_{ij} = (\varphi_i, \varphi_j).$$

   iii) Prove the following statement by counter example. $V = C([a,b])$ equipped with $\|\cdot\|_V = \|\cdot\|_\infty$ is not strictly normed.

   4 points

3. submission until 06.05.2024
   Approximation of functions by polygonal chains, programming exercise. Take the function $f(x) = \sin(x)$ on the interval $[0, 2\pi]$. Subdivide the interval into n equidistant sub intervals of step size $h = 2\pi/n$ and use
   $$S_n = \left\{u_n \in C([0,2\pi])\ :\ u_n|_{[kh,(k+1)h]} \in P_1([kh,(k+1)h]),\ k = 0,\dots,n-1\right\}$$
   as space for approximation.

   i) Determine the error
   $$\max_{k=0,\dots,n} |f(kh) - u_n(kh)| \approx \|f - u_n\|_\infty$$
   for $n = 2^l$, $n = 0, 1, \dots 128$.

   ii) Which functional dependence can you observe between the error and the step size?

   6 points

The exercises should be solved in groups of two students. They have to be submitted until Sie **Monday, 29.04.2024, 10:00**, either in the box of the tutor or electronically via whiteboard.