Fachbereich Mathematik und Informatik/ Institut für Mathematik
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Numerik I

English translation of Übungsserie 01

Attention: Only solutions which provide a comprehensible reasoning will be graded. Every statement has to be argued. You can use results from the lecture. Statments without reasoning won't get any points.

1. Let X be a normed vector space.

   a) Proof the following statements. The space X is an inner product space (Prä-Hilbert-Raum), if and only if the following parallelogram law is valid

   $$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2 \quad \forall\, x, y \in X.$$

   Hints: You are allowed to use the polarization identity, i.e. given the apropriate scalar product you are allowed to assume

   $$4(x,y) = \|x+y\|^2 - \|x-y\|^2.$$

   For linearity one seperately shows the additivity and the homogeneity. To this end it can be helpful to first proof $(x+\delta x, y) + (x - \delta x, y) = 2(x,y)$ and then argue homogeneity as a special case thereof. This argument can be extended up to the rational numbers. To conclude the statement for the reals you are allowed to use the continuity of the norm without further proof.

   b) Every inner product space is strictly convex i.e.

   $$\frac{1}{4}\|x+y\|^2 < 1 \quad \forall\, x, y \in X \text{ mit } x \neq y, \|x\| = \|y\| = 1.$$

5 points

2. The Chebyshev polynomials of first kind are given by

   $$T_n(x) = \cos\left(n \arccos(x)\right), \quad n = 0, 1, 2, \ldots.$$

   a) Compute all zeros of these polynomials.

   b) The scalar product of $L^2(-1,1)$ is given by

   $$(u,v) = \int_{-1}^{1} u(x)v(x)\ dx.$$

   Proof the following statement. The Chebyshev polynomials of first kind do not define an orthogonal system with respect to the mentioned scalar product.

c) Now proof that the Chebyshev polynomials of first kind are indeed an orthogonal system with respect to a weighted inner product given by

$$(u, v)_w = \int_{-1}^{1} \frac{u(x)v(x)}{\sqrt{1-x^2}}\ dx.$$

5 points

The exercises should be solved in groups of three or four students. They have to be submitted until Sie **Monday, 25.04.2022, 12:00**, either in the box of the tutor or electronically.