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Numerical Mathematics II

Exercise Problems 11

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Consider the linearly implicit Runge–Kutta method `ode23s`

$$\begin{aligned}(I - ahJ)\,K_1 &= f(y_k), \quad a = \frac{1}{2+\sqrt{2}},\\ (I - ahJ)\,K_2 &= f\left(y_k + \frac{1}{2}hK_1\right) - ahJK_1,\\ y_{k+1} &= y_k + hK_2\end{aligned}$$

   with $J = f_y(y_k) = f'(y_k)$. Show that the stability function of this method, for sufficiently small step sizes h, is

$$R(z) = \frac{1 + (1-2a)z}{(1-az)^2}.$$

   Hint. It suffices to consider an autonomous equation. Then, one has to apply the method to the usual model initial value problem.

2. Consider the problem and the discretization from Exercise Problems 06, problem 3 (c)–(d). Use grids with $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$ and the parameter $\varepsilon = 0.01$.

   (a) Use the MATLAB build-in routine `bicgstab` for solving these problems.

   (b) Implement BiCGStab and apply this method for solving these problems.

   Stop the iterations if the norm of the residual vector is less than 10^{-10}. Give the number of iterations for both methods.

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Jan. 22, 2013** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.