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Numerical Mathematics II

Exercise Problems 10

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Compute the stability function of the classical Runge–Kutta method (Example 1.33) and draw a sketch of the domain of stability.
   Hint: For drawing the domain of stability, one can compute sufficiently many values of the stability function, which can be performed quickly with a short program. The domain of stability is simply connected.

2. Write a MATLAB code that solves the model initial value problem for stability, problem (2.24) with the explicit Euler method, the implicit Euler method, and the trapezoidal rule. Take $\lambda = -10$, the interval $[0, 1]$, and meshes with 2, 4, 8, 16 intervals. Compute the error at the first node $x_1 = h$ and at the final point $x = 1$.
   Hint: Use the formulas from Example 2.53 for implementing the methods.

3. The class of so-called M-matrices will become important in the lecture.
   A matrix $A \in \mathbb{R}^{n\times n}$ is called M-matrix if it satisfies the following conditions

   1. $a_{ij} \leq 0$ for $i, j = 1, \dots, n$, $i \neq j$,
   2. A is non-singular and A^{-1} is non-negative, i.e. $a_{ij} \geq 0$ for $i, i = 1, \dots, n$.

   Prove the following statement: An M-matrix possesses positive diagonal entries.

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Jan. 15, 2013** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.