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## Numerical Mathematics II

### Exercise Problems 08

**Attention:** The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Show the properties of the matrix exponential given in Lemma 2.37, lecture notes *Numerical Methods for Ordinary Differential Equations*.
2. Solve the following initial value problem

$$\begin{aligned}y'(x) + 3y(x) + z(x) &= xe^{-2x}, \quad y(0) = z(0) = 1 \\z'(x) - y(x) + z(x) &= x^2.\end{aligned}$$

3. Continue Problem 4 from the Exercise Problems 03. In this problem, iterative schemes had to be implemented for solving a linear system of equations with symmetric positive definite matrix. Consider again the meshes with  $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$ .
  - (a) Implement the Conjugate Gradient method as given in Algorithm 6.8. How does the number of iteration change with  $h$ ?
  - (b) Use the Conjugate Gradient method as provided by MATLAB (`pcg`). Compare the number of iterations with your own implementation.
  - (c) Measure the executing times for both methods with the commands `tic` and `toc`. Which implementation is more efficient?

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Dec. 18, 2012** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.