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Numerical Mathematics II

Exercise Problems 06

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Find the general solutions of the following ODEs:

$$\begin{array}{lrcl} (a) & y''(x) + 3y'(x) + 2y(x) & = & 1/(e^{-x} + e^{-2x}), \\ (b) & y''(x) + 6y'(x) + 9y(x) & = & e^{-3x}/(1+x). \end{array}$$

2. Prove the following statement: The homogeneous linear ODE of order n can be reduced to the solution of a homogeneous linear ODE of order $(n-1)$ by applying the ansatz $g(x)y_0(x)$, where $y_0(x)$ is a known solution of the homogeneous equation.
   Hint: Choose $g(x)$ such that $g(x)y_0(x)$ is another solution of the homogeneous equation.

3. Continue Problem 4 from Exercise Problems 05.

   (a) Consider a decomposition of $[0,1]$ by a grid as, e.g., in Problem 2 from Exercise Problems 01. Show that the approximation (backward finite difference)

$$u'(x_i) \approx \frac{u(x_i) - u(x_{i-1})}{h} = u_{\bar{x},i}, \quad i = 1, \ldots, n-1,$$

   is of first order, i.e.

$$u_{\bar{x},i} = u'(x_i) + \mathcal{O}(h)$$

   if $u \in C^2([0,1])$.

   (b) Modify the code of Problem 4 from Exercise Problems 05 such that it applies to the differential equation given here, where the first order derivative is approximated by the backward difference.

   (c) Consider the grids with $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$ and compute the solution for $\varepsilon \in \{1, 10^{-2}, 10^{-4}, 10^{-6}\}$ (solve the linear system of equations with the backslash command) and compute the errors $\|u - u_h\|_{l^2}$. How does the error behaves with respect to the size ε ?

   (d) Consider the same situations as in Problem 3c. Solve the arising linear systems of equations with the Jacobi method, the Gauss–Seidel method, and GMRES. Use the GMRES routine provided by MATLAB (read the documentation carefully). GMRES should be used without restart, with $tol = 1e-10$, and the maximal number of iterations should coincide with the dimension of the problem. Give the number of iterations for all methods ? How do they change with respect to the size of ε ?

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Dec. 4, 2012** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.