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Numerical Mathematics II

Exercise Problems 05

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Assume that the functions x^3 and x^4 are linearly independent solutions of a homogeneous linear differential equation of second order. Find such an equation.

2. Prove Theorem 2.22 from the Numerical Methods for ODEs.

3. Find the general solutions of

$$\begin{array}{lrcl} (a) & y''(x) + 4y(x) & = & e^{2x}\sin(x) + \cos(2x), \\ (b) & y''(x) + 2y'(x) + 10y(x) & = & 20x^3 + 17x^2 + 14x - 9. \end{array}$$

   Hint : use an appropriate ansatz to find a special solution of the inhomogeneous equation.

4. Consider the following differential equation

$$-\varepsilon u''(x) + u'(x) = 1,$$

   where $\varepsilon > 0$ is a parameter.

   (a) Find the general solution of this equation.

   (b) Fix the constants of the general solution by using the (boundary) conditions

$$u(0) = u(1) = 0.$$

   (c) Draw the solution in $[0, 1]$ for $\varepsilon \in \{1, 10^{-2}, 10^{-4}, 10^{-6}\}$. How does the solution change ?

   (d) Consider a decomposition of $[0, 1]$ by a grid as, e.g., in Problem 2 from Exercise Problems 01. Show that the approximation (central finite difference)

$$u'(x_i) \approx \frac{u(x_{i+1}) - u(x_{i-1})}{2h} = u_{x,i}, \quad i = 1, \ldots, n-1$$

   is of second order, i.e.

$$u_{x,i} = u'(x_i) + \mathcal{O}(h^2)$$

   if $u \in C^3([0, 1])$.

   (e) Modify the code of Problem 2 from Exercise Problems 02 such that it applies to the differential equation given here, where the first order derivative is approximated by the central difference.

(f) Consider the grid with $h = 1/128$ and compute the solution for $\varepsilon \in \{1, 10^{-2}, 10^{-4}, 10^{-6}\}$ (solve the linear system of equations with the backslash command), compute the errors $\|u - u_h\|_{l^2}$, and draw the computed solutions. How do they change when ε becomes smaller ?

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Nov. 27, 2012** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.