Department for Mathematics and Computer Science
Free University of Berlin
Prof. Dr. V. John, john@wias-berlin.de
Hanne Hardering, harderin@zedat.fu-berlin.de

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Numerical Mathematics II

Exercise Problems 03

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Prove Theorem 2.9.

2. Solve the following ordinary differential equations

$$\begin{array}{lrcl} \text{(a)} & y'(x) + y^2(x) & = & 1, \\ \text{(b)} & y'(x) + y(x)\cos x & = & 0, \\ \text{(c)} & y'(x) & = & xy^7(x), \\ \text{(d)} & 2y(x)y'(x) & = & x^2. \end{array}$$

3. Solve the following initial value problem

$$xy'(x) + 3y(x) = x^2\ , \quad x_0 = 0, y_0 = 0.$$

4. Continue Problem 2 from the Exercise Problems 02. From Friday Nov. 9, a sample code for Problem 2 from Exercise Problems 02 will be available on the homepage of the class. You can use this sample code as basis for solving the problems given below.

   (a) Compute the spectral condition number for the matrices that are obtained with $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$. What can be observed?

   (b) Implement the damped Jacobi method for solving the linear systems of equations that are obtained for $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$. Use the damping factors $\omega \in \{0.1, 0.2, \ldots, 1, 1.1\}$.

   Use as starting iterate the zero vector and stop the iteration if the Euclidean norm of the residual $\|A\mathbf{u} - \mathbf{f}\|_2$ is less than $1e-10$ or after 100 000 iterations were performed. Count the number of iterations. What can be observed?

   (c) Implement the SOR method for solving the linear systems of equations that are obtained for $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$. Use the damping factors $\omega \in \{0.1, 0.2, \ldots, 1.8, 1.9, 2.0\}$.

   Use as starting iterate the zero vector and stop the iteration if the Euclidean norm of the residual $\|A\mathbf{u} - \mathbf{f}\|_2$ is less than $1e-10$ or after 10 000 iterations were performed. Count the number of iterations. Give for each h the best relaxation factor (least number of iterations)!

   The simulations may take a while. The best way is to write a loop that performs one simulation after the other.

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Nov. 13, 2012** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.