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Berlin, 30.10.2012

Numerical Mathematics II

Exercise Problems 02

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Solve the following problems.

   (a) Let G_{GS} be the iteration matrix of the Gauss–Seidel method. Show that

   $$G_{\text{GS}} = -D^{-1}\left(LG_{\text{GS}} + U\right).$$

   (b) Consider the SOR method and verify the following identities

   $$\begin{aligned} D + \omega L &= \left(1 - \frac{\omega}{2}\right) D + \frac{\omega}{2} A + \frac{\omega}{2}\left(L - U\right), \\ \left(1 - \omega\right) D - \omega U &= \left(1 - \frac{\omega}{2}\right) D - \frac{\omega}{2} A + \frac{\omega}{2}\left(L - U\right). \end{aligned}$$

2. Consider Problem 2 from the Exercise Problems 01.

   (a) Solve the boundary value problem given there for

   $$f(x) = -6\pi\cos(3\pi x) + 9\pi^2 x \sin(3\pi x),$$

   and $a = b = 0$.

   (b) Solve this problem numerically using the discretization described in Exercise Problems 01 with $h \in \{1/8, 1/16, 1/32, 1/64, 1/128, 1/256\}$. One can use the backslash command in MATLAB. Give the error of the computed solution u_h to the analytic solution u in the following norm

   $$\|u - u_h\|_{l^2} = \left(\frac{1}{N-1}\sum_{i=1}^{N-1}\left(u(x_i) - u_i\right)^2\right)^{1/2},$$

   where N is the number of nodes.

   (c) Use the following ansatz of the convergence order

   $$\|u - u_h\|_{l^2} = ch^{\alpha}.$$

   Compute α by using the results on the two finest grids.

   (d) Replace the direct solver of the linear system of equations by the Jacobi method. Use as starting iterate the zero vector and stop the iteration if the Euclidean norm of the residual $\|A\mathbf{u} - \mathbf{f}\|_2$ is less than $1e-10$. Count the number of iterations. What can be observed?

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Nov. 06, 2012** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.