Department for Mathematics and Computer Science
Free University of Berlin
Prof. Dr. V. John, john@wias-berlin.de
Hanne Hardering, harderin@zedat.fu-berlin.de

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Numerical Mathematics II

Exercise Problems 01

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Consider an autonomous initial value problem

$$y'(x) = f(y(x)), \quad y(x_0) = y_0.$$

This problem shall be solved on an equidistant grid with a 3 stage Runge–Kutta method. Derive the conditions for this method of being of third order, see Remark 1.28.
Hint. Proceed as in Example 1.29.

2. Consider the differential equation (Poisson equation, boundary value problem)

$$\begin{array}{rcll} -u'' & = & f & \text{in } (0,1), \\ u(0) & = & a, & \\ u(1) & = & b. & \end{array}$$

Use an equidistant grid

$$0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1, \quad h = x_i - x_{i-1},\ i = 1, \ldots, n,$$

for the discretization of the second derivative.

(a) Show that the approximation (finite difference)

$$u''(x_i) \approx \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1})}{h^2} = u_{xx,i}, \quad i = 1, \ldots, n-1$$

is of second order, i.e.

$$u_{xx,i} = u''(x_i) + \mathcal{O}(h^2)$$

if $u \in C^4([0,1])$.

(b) Insert the approximation of the second order derivative into the differential equation and derive a linear system of equations for the values $u_i = u(x_i)$, $i = 0, \ldots, n$. As approximation for the right hand side take $f_i = f(x_i)$, $i = 0, \ldots, n$.

(c) Reduce this linear system of equations to a system for u_i, $i = 1, \ldots, n-1$, by taking into consideration the boundary conditions.

(d) Given n. Write a MATLAB code that computes the matrix of this system and stores this matrix in `sparse` format.

This matrix is needed for further exercise problems.

The exercise problems should be solved in groups of two students. The written parts have to be submitted until **Tuesday, Oct. 30, 2012** either before one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.