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Numerical Mathematics IV

Exercise Problems 04

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Write a code, e.g., in MATLAB, which approximates the solution the two-point boundary value problem from Example 2.8 (the standard example).

   - Use as diffusion coefficient $\varepsilon \in \{1, 10^{-3}, 10^{-6}, 10^{-9}\}$.
   - Apply the following methods:
     - the central difference scheme,
     - the simple upwind scheme,
     - the Samarskij upwind scheme,
     - the Iljin–Allen–Southwell scheme.
   - The simulations should be performed on grids with

     $$N \in \{4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048\}$$

     intervals. Compute the error to the solution of the continuous problem in the discrete maximum norm. Based on the error, compute the numerical order of convergence k.
     A formula for k can be obtained from the ansastz

     $$\|u - u_h\|_{\infty,d} = ch^k,$$

     by solving this ansatz for k for the mesh widths h and $2h$ (with the same constant c).

   16 points

2. Extend the code for the simple upwind scheme to a Shishkin mesh.

   - The problems to be solved are the same as in Problem 1.
   - The transistion points are defined by $\sigma \in \{0.5, 1, 2, 4\}$.

   Hint: It is simpler to program the simple upwind scheme in the form with the backward finite difference. **8 points**

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Jan. 20, 2014** by email. Please send the codes and the tables with the errors and orders of convergence.