Department for Mathematics and Computer Science
Free University of Berlin
Prof. Dr. V. John, john@wias-berlin.de

Berlin, 05.01.2014

Numerical Mathematics IV

Exercise Problems 03

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Solve the following problems.
   (a) Let $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ be an M-matrix. Show that $a_{ii} > 0$, $i = 1, \ldots, n$.
   (b) Show that the discrete solution of Example 3.40 possesses the given form.
   (c) Show that the functions
   $$\sigma(q) = \max\{1, q\}, \quad \sigma(q) = \sqrt{1+q^2}, \quad \sigma(q) = 1 + \frac{q^2}{1+q}.$$
   satisfy the assumptions of Theorem 3.47.

6 points

2. Solve the following problems.
   (a) Show that the bound $\eta_0(\nu)$ in Theorem 5.16 behaves like ν^{-1}.
   (b) Consider P_1 finite elements on an equidistant grid in one dimension. Prove the norm equivalence (5.12).
   Hint: Consider the type of functions which has to be integrated and how this integration can be performed exactly.

6 points

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Jan. 13, 2014** either by email or in one of the classes.