Department for Mathematics and Computer Science
Free University of Berlin
Prof. Dr. V. John, john@wias-berlin.de

Berlin, 02.12.2013

Numerical Mathematics IV

Exercise Problems 02

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

1. Solve the following problems.

   (a) Show that the eigenvectors of A are also the eigenvectors of $S_{\mathrm{jac},\omega}$ (Remark 3.9).

   (b) Show that the damped Jacobi method converges fastest for the model problem if $\omega = 1$, i.e., solve the min-max problem at the end of the proof of Theorem 3.10.

   4 points

2. Show that Green's function given in Theorem 2.26 satisfies the jump condition

   $$\partial_x \Gamma(x+0,x) - \partial_x \Gamma(x-0,x) = -1, \quad \text{for } x \in (0,1).$$

   4 points

3. The use of higher order finite element discretizations leads usually too much more accurate results with less degrees of freedom than the use of first order discretizations. However, the implementation of higher order finite elements is more involved and in many situations, the solution of the arising systems of equations is more complicated. In the lecture notes, Table 2.1, and in the Exercise Problems 01, studies for the Poisson equation discretized with the P_1 finite element were performed. For this problem, the Poisson equation was discretized with the P_3 finite element in two dimensions.

   Load the files provided on the homepage of this course. One of the files is the executable `rsh1_2d` (Linux, 64 bit) which solves the problem

   $$-\Delta u = 1 \quad \text{in } (0,1)^2$$

   with homogeneous boundary conditions. The files

   `rhs1_ssor_p3.dat, rhs1_cg_p3.dat, rhs1_mg_p3.dat, rhs1_fgmres_mg_p3.dat, rhs1_umfpack_p3.dat`

   contain the parameters for solving this problem with different solvers. Execute `chmod a+x ./rhs1_2d` and then the code runs by

   ```
   ./rsh1_2d rhs1_ssor_p3.dat
   ```

   and so on. One will get an output file with the name `rhs1_ssor_p3.out` and so on.

   Compile a table of the form of Table 2.1 from the lecture notes for this problem. The computing times for each level of refinement can be obtained with

   ```
   grep 'time for linear problem' *.out
   ```

   For the number of iterations, one has to check each output file individually. Interprete the obtained results! **4 points**

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Dec. 09, 2013** either by email or in one of the classes.