Lösungen zum 33. Präsenzblatt für MfI 3

1. Aufgabe :
   Satz von Taylor:

$$f(x,y) = \sum_{k=0}^{m} \frac{1}{k!} \left(\sum_{i=0}^{k} \left(\binom{k}{i} \right) (x-x_0)^i (y-y_0)^{k-i} \frac{\partial^k f}{\partial x^i \partial y^{k-i}}(x_0,y_0) \right) + R(x,y)$$

			(x_0,y_0)
$f(x,y)$	$=$	$\ln(1+x+y)$	$\ln(1)=0$
$f_x(x,y)$	$=$	$\frac{1}{1+x+y}$	1
$f_y(x,y)$	$=$	$\frac{1}{1+x+y}$	1
$f_{xx}(x,y)$	$=$	$-\frac{1}{(1+x+y)^2}$	-1
$f_{xy}(x,y)$	$=$	$-\frac{1}{(1+x+y)^2}$	-1
$f_{yy}(x,y)$	$=$	$-\frac{1}{(1+x+y)^2}$	-1
$f_{xxx}(x,y)$	$=$	$\frac{2}{(1+x+y)^3}$	2
	$=$	$f_{xxy}(x,y),\ f_{xyy}(x,y),\ f_{yyy}(x,y)$	
$f_{xxxx}(x,y)$	$=$	$-\frac{6}{(1+x+y)^4}$	-6
	$=$	alle anderen Ableitungen	

$$\begin{aligned} \Longrightarrow f(x,y) &\approx x+y+\frac{1}{2}\left(-x^2-2xy-y^2\right)+\frac{2}{6}\left(x^3+3x^2y+3xy^2+y^3\right) \\ &\quad +\frac{-6}{24}\left(x^4+4x^3y+6x^2y^2+4xy^3+y^4\right) \\ &= (x+y)-\frac{1}{2}(x+y)^2+\frac{1}{3}(x+y)^3-\frac{1}{4}(x+y)^4 \end{aligned}$$

2. Aufgabe :

$$\begin{aligned} u_x &= \frac{\partial g}{\partial r}\frac{\partial r}{\partial x}+\frac{\partial g}{\partial \varphi}\frac{\partial \varphi}{\partial x} \\ &= g_r r_x + g_\varphi \varphi_x \\ u_{xx} &= (u_x)_x \\ &= \frac{\partial g_r}{\partial r}\frac{\partial r}{\partial x}r_x+\frac{\partial g_r}{\partial \varphi}\frac{\partial \varphi}{\partial x}r_x+g_r\frac{\partial r_x}{\partial x}+\frac{\partial g_\varphi}{\partial r}\frac{\partial r}{\partial x}\varphi_x+\frac{\partial g_\varphi}{\partial r}\frac{\partial \varphi}{\partial x}\varphi_x+g_\varphi\frac{\partial \varphi_x}{\partial x} \\ &= g_{rr}r_x^2+g_{r\varphi}\varphi_x r_x+g_r r_{xx}+g_{\varphi r}r_x\varphi_x+g_{\varphi\varphi}\varphi_x^2+g_\varphi\varphi_{xx} \end{aligned}$$