Lösungen zum 32. Präsenzblatt für MfI 3

1. Aufgabe :

(a)

$$
\begin{aligned}
(\alpha \mathbf{f}+\beta \mathbf{g})'(\xi) &= J(\alpha \mathbf{f}+\beta \mathbf{g})(\xi) \\
&= \begin{pmatrix} \frac{\partial}{\partial x_1}(\alpha \mathbf{f}+\beta \mathbf{g})_1(\xi) & \cdots & \frac{\partial}{\partial x_n}(\alpha \mathbf{f}+\beta \mathbf{g})_1(\xi) \\ \vdots & & \vdots \\ \frac{\partial}{\partial x_1}(\alpha \mathbf{f}+\beta \mathbf{g})_m(\xi) & \cdots & \frac{\partial}{\partial x_n}(\alpha \mathbf{f}+\beta \mathbf{g})_m(\xi) \end{pmatrix} \\
&= \begin{pmatrix} \alpha\frac{\partial f_1}{\partial x_1}(\xi)+\beta\frac{\partial g_1}{\partial x_1}(\xi) & \cdots & \alpha\frac{\partial f_1}{\partial x_n}(\xi)+\beta\frac{\partial g_1}{\partial x_n}(\xi) \\ \vdots & & \vdots \\ \alpha\frac{\partial f_m}{\partial x_1}(\xi)+\beta\frac{\partial g_m}{\partial x_1}(\xi) & \cdots & \alpha\frac{\partial f_m}{\partial x_n}(\xi)+\beta\frac{\partial g_m}{\partial x_n}(\xi \end{pmatrix} \\
&= \alpha\, J\mathbf{f}(\xi)+\beta\, J\mathbf{g}(\xi) \\
&= \alpha\, \mathbf{f}'(\xi)+\beta\, \mathbf{g}'(\xi)
\end{aligned}
$$

(b)

$$
\begin{aligned}
(J)_{ij} &= \frac{\partial f_i}{\partial x_j} \\
J &= \begin{pmatrix} y & x \\ y\sinh(xy) & x\sinh(xy) \\ \frac{2x}{1+x^2} & 0 \end{pmatrix}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\left(\sqrt{xy}+xy\frac{1}{2}(xy)^{-\frac{1}{2}}\right)e^{x^2y}-x\sqrt{xy}e^{x^2y}2xy}{e^{2x^2y}} \\
&= \frac{\sqrt{xy}+\frac{1}{2}\sqrt{xy}-2x^2y\sqrt{xy}}{e^{x^2y}} \\
&= \frac{\frac{3}{2}\sqrt{xy}-2x^2y\sqrt{xy}}{e^{x^2y}} \\
\frac{\partial f}{\partial y} &= \frac{\frac{1}{2}x^2(xy)^{-\frac{1}{2}}e^{x^2y}-x\sqrt{xy}e^{x^2y}x^2}{e^{2x^2y}} \\
&= \frac{1}{2\sqrt{xy}}\,\frac{x^2-2x^4y}{e^{x^2y}} \\
\nabla f &= \left(\frac{\frac{3}{2}\sqrt{xy}-2x^2y\sqrt{xy}}{e^{x^2y}},\ \frac{1}{2\sqrt{xy}}\,\frac{x^2-2x^4y}{e^{x^2y}}\right) \\
\nabla f(-2,-3) &= (1.0166e+07, 3.3222e+06)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{a} &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \\
|\mathbf{a}| &= \sqrt{5} \\
\nabla f\,(-2,-3)\cdot\frac{\mathbf{a}}{\sqrt{5}} &= 1.0578e+07
\end{aligned}
$$

2. Aufgabe :

$$
\begin{aligned}
g(\mathbf{x}) &= \left|\left\langle \mathbf{x}^0, \mathbf{x}\right\rangle\right|^2 \\
&= \left|\sum_{i=1}^{m} x_i^0 x_i\right|^2 \\
&= \left(\sum_{i=1}^{m} x_i^0 x_i\right)^2 \\
\frac{\partial g(\mathbf{x})}{\partial x_i} &= 2\left(\sum_{i=1}^{m} x_i^0 x_i\right) x_i^0 \\
\Longrightarrow \qquad \nabla g(\mathbf{x}) &= 2\left\langle \mathbf{x}^0, \mathbf{x}\right\rangle \left(\mathbf{x}^0\right)^T
\end{aligned}
$$