Lösungen zum 28. Präsenzblatt für MfI 3

1. Aufgabe :

$$u(x) \quad = \quad \begin{cases} a & 0 < x < \frac{\pi}{2} \\ -a & \frac{\pi}{2} < x < \frac{3\pi}{2} \\ a & \frac{3\pi}{2} < x < 2x \end{cases}$$

Berechnung der Koeffizenten:

$$\begin{aligned}
a_0 &= 0 \\
a_k &= \frac{1}{\pi}(u, \cos(kx)) \\
&= \frac{1}{\pi}\int_0^{2\pi} u(x)\,\cos(kx)\,\mathrm{d}x \\
&= \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}} a\,\cos(kx)\,\mathrm{d}x + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -a\,\cos(kx)\,\mathrm{d}x + \int_{\frac{3\pi}{2}}^{2\pi} a\,\cos(kx)\,\mathrm{d}x\right] \\
&= \frac{1}{\pi}\left[a\left[\frac{1}{k}\sin(kx)\right]_0^{\frac{\pi}{2}} - a\left[\frac{1}{k}\sin(kx)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + a\left[\frac{1}{k}\sin(kx)\right]_{\frac{3\pi}{2}}^{2\pi}\right] \\
&= \frac{1}{\pi}\left[\frac{a}{k}\sin(k\frac{\pi}{2}) - \frac{a}{k}\sin(k\frac{3\pi}{2}) + \frac{a}{k}\sin(k\frac{\pi}{2}) - \frac{a}{k}\sin(k\frac{3\pi}{2})\right] \\
&= \frac{1}{\pi}\left[\frac{2a}{k}\sin(k\frac{\pi}{2}) - \frac{2a}{k}\sin(3k\frac{\pi}{2})\right] \\
&= \frac{2a}{\pi k}\left(\sin(k\frac{\pi}{2}) - \sin(3k\frac{\pi}{2})\right) \\
&= \frac{4a}{\pi k}\sin(k\frac{\pi}{2}) \\
&= \pm\frac{4a}{\pi k}, \quad \text{für ungerade } k \\
&= 0, \quad \text{für gerade } k \\
b_k &= \frac{1}{\pi}(u, \sin(kx)) \\
&= \frac{1}{\pi}\int_0^{2\pi} u(x)\,\sin(kx)\,\mathrm{d}x \\
&= \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}} a\,\sin(kx)\,\mathrm{d}x + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -a\,\sin(kx)\,\mathrm{d}x + \int_{\frac{3\pi}{2}}^{2\pi} a\,\sin(kx)\,\mathrm{d}x\right] \\
&= \frac{1}{\pi}\left[a\left[-\frac{1}{k}\cos(kx)\right]_0^{\frac{\pi}{2}} - a\left[-\frac{1}{k}\cos(kx)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + a\left[-\frac{1}{k}\cos(kx)\right]_{\frac{3\pi}{2}}^{2\pi}\right]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{\pi}\left[a\left(\frac{1}{k}-\frac{1}{k}\cos(k\frac{\pi}{2})\right)-a\left(\frac{1}{k}\cos(k\frac{\pi}{2})-\frac{1}{k}\cos(k\frac{3\pi}{2})\right)+a\left(\frac{1}{k}\cos(k\frac{3\pi}{2})-\frac{1}{k}\cos(2k\pi)\right)\right] \\
&= \frac{2a}{\pi k}\left(\cos(3k\frac{\pi}{2})-\cos(k\frac{\pi}{2})\right) \\
&= 0
\end{aligned}
$$

Fourierreihe:

$$
u(x) = \sum_{k=0}^{\infty} a_k \cos(kx)
$$

2. Aufgabe :

$$
\begin{aligned}
A &= \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \\
A^{-1} &= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \\
\det(A) &= 1 \\
A &= \begin{pmatrix} 0 & 1 & 1/\sqrt{2} \\ 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} \end{pmatrix} \\
A^{-1} &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \\
\det(A) &= -\frac{1}{2}\sqrt{2} \\
A &= \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \\
A^{-1} &= \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \\
\det(A) &= 1 \\
A &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{3} & -1/2 & 0 \\ 0 & 1/\sqrt{3} & 0 & 1 \\ 0 & 1/\sqrt{3} & 1/2 & 0 \end{pmatrix} \\
A^{-1} &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & 0 & \sqrt{3}/2 \\ 0 & -1 & 0 & 1 \\ 0 & -1/2 & 1 & -1/2 \end{pmatrix} \\
\det(A) &= -\frac{1}{3}\sqrt{3}
\end{aligned}
$$