Generation of Multivariate Scenario Trees to Model Stochasticity in Power Management

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***Abstract*— Modern electricity portfolio and risk management models represent multistage stochastic programs. The input of such programs consists in a finite set of scenarios having the form of a scenario tree. They model the probabilistic information on random data (electrical load, stream flows to hydro units, market prices of fuel and electricity). Since the corresponding deterministic equivalents of multistage stochastic programs are mostly large scale, one has to find significant tree-structured scenarios. Our approach to generate multivariate scenario trees is based on recursive deletion and bundling of scenarios out of some given (possibly large) scenario set originating from historical or simulated data. The procedure makes use of certain Monge-Kantorovich transportation distances for multivariate probability distributions. We report on computational results for generating load-inflow scenario trees based on realistic data of EDF Electricité de France.**

***Index Terms*— Stochastic programming, power management, scenario reduction, scenario tree construction**

I. Introduction

The operation of power systems under deregulated market conditions leads to an increasing interest in incorporating uncertainty and risk into optimization models (see [12], [33], [34]). The corresponding stochastic optimization models require decisions on the basis of given probabilistic information on random data. Typically, such models use a finite number of scenarios to model uncertainty of relevant data, e.g., [1], [3], [9], [10], [14], [30], [32]. Each scenario can be viewed as one realization of a certain multi-dimensional stochastic data process of the model. All scenarios and their probabilities represent an approximation of the probability distribution given by the random data. Clearly, a good approximation may involve a very large number of scenarios. But, due to computational complexity for most practical problems, the number of scenarios must be restricted to have the continuing ability to solve the stochastic model. The main challenge of scenario tree generation is to effect a compromise between a good approximation of the probability distribution and the dimension of the stochastic model.

During the last few years scenario modelling and the generation of scenario trees became a very active field of research in stochastic programming (see the survey [6]). Recently, existing methods and new techniques were refined and proposed, respectively, for generating scenario trees for multistage models. We mention here in chronological order

(i) *bound-based approximation* methods [11], [8], [4],
(ii) *Monte Carlo-based* sampling schemes [21], [5], [2], [31],
(iii) the *moment-matching principle* [19], [20],
(iv) optimal *approximations based on probability metrics* [25], [18], [7], [13], and
(v) the use of *integration quadratures* [23].

A systematic comparison, both theoretical and computational, of these approaches has not been undertaken so far. Some principles for the evaluation of scenario tree generation methods are presented in [22]. Potential shortcomings of the approach (iii) are discussed in [18].

We propose a technique that belongs to the group (iv) and is based on probability metrics that are associated with the stability of the underlying stochastic program. The input of the method consists in a finite number of scenarios that are provided by the user and, say, are obtained from historical data by data analysis and resampling techniques or from statistical models calibrated to the relevant historical data. Then the method constructs a scenario tree by recursive (optimal) scenario reduction [13], [14].

II. Generation of scenario trees

A. Approximation of stochastic programs

The recent paper [28] surveys quantitative stability results for stochastic programs. It is shown there that the distances $\hat{\mu}_r$, $r \geq 1$, of multivariate probability distributions given by Monge-Kantorovich (mass) transportation problems [27] are relevant for the stability of two-stage models. More precisely, $\hat{\mu}_1$ is relevant if either right-hand sides or prices are stochastic and $\hat{\mu}_2$ is important if both are stochastic. In case of multistage stochastic programs the distances $\hat{\mu}_r$ as well as a functional measuring the 'distance' of the information structures are indispensable for stability [17]. In the present paper, we concentrate on the functionals $\hat{\mu}_r$. The effects of distances of information structures are discussed in [17] and the forthcoming paper [16].

Let us consider the important case that both, the original and approximate probability distribution P and Q, respectively, have a finite support, i.e., a finite number of scenarios in some Euclidean space $\mathbb{R}^s$. Let the supports be given by

$$\text{supp}(P) = \{\xi^1, \dots, \xi^N\}, \ \text{supp}(Q) = \{\tilde{\xi}^1, \dots, \tilde{\xi}^M\},$$

and the probabilities by

$$p_i = P(\{\xi^i\}) \quad \text{and} \quad q_j = Q(\{\tilde{\xi}^j\}).$$

The Monge-Kantorovich functional $\hat{\mu}_r$ for $r \geq 1$ is defined as

$$\hat{\mu}_r(P,Q) = \inf \left\{ \sum_{i=1}^{N} \sum_{j=1}^{M} c_r(\xi^i, \tilde{\xi}^j)\eta_{ij} \,\middle|\, \eta_{ij} \geq 0, \right. \quad (1)$$
$$\left. \sum_{l=1}^{M} \eta_{il} = p_i, \sum_{l=1}^{N} \eta_{lj} = q_j, i = 1,\ldots,N, j = 1,\ldots,M \right\}.$$

Thus, the functional $\hat{\mu}_r$ represents the optimal value of a *linear transportation problem.* The non-negative cost function c_r is defined by

$$c_r(\xi, \tilde{\xi}) = \max\{1, ||\xi - \xi_0||, ||\tilde{\xi} - \xi_0||\}^{r-1} ||\xi - \tilde{\xi}||, \quad (2)$$

where ξ_0 is some fixed element in $I\!R^s$. The cost function can be viewed as a certain distance function on the set of all scenarios. In case $r = 1$ the cost function c_r coincides with the metric induced by the norm on $I\!R^s$.

B. Optimal scenario reduction

The scenario reduction approach was first developed in [7] and enhanced in [15]. Let us recall its main ideas: We consider a discrete distribution P with scenarios ξ^i and probabilities p_i, $i = 1,\ldots,N$, and another discrete distribution Q supported by a subset of scenarios ξ^j, $j \in \{1,\ldots,N\} \setminus J$, of P and probabilities q_j, $j \notin J$, i.e, the index set J describes the set of deleted scenarios. The main result in [7] provides the best possible distance $\hat{\mu}_r(P,Q)$ if the index set J is fixed, but the weights q_j vary. The optimal distribution Q^* is given by the *optimal redistribution*

$$q_j^* = p_j + \sum_{i \in J_j} p_i, \; j \notin J, \quad (3)$$

where the index sets J_j are given by

$$J_j = \{i \in J : j = j(i)\}$$

and $j(i)$ is a selection of the index set of closest scenarios to ξ^i

$$j(i) \in \arg\min_{j \notin J} c_r(\xi^i, \xi^j), \; i \in J.$$

Hence, the optimal scenario reduction consists in adding each deleted scenario weight to that of some of those scenarios being closest with respect to c_r. The distance of P and Q^* is given by

$$D_J := \hat{\mu}_r(P, Q^*) = \sum_{i \in J} p_i \min_{j \notin J} c_r(\xi^i, \xi^j). \quad (4)$$

To determine an optimal index set J^* with a prescribed number of $N-n$ elements, one has to solve the combinatorial optimization problem

$$J^* \in \arg\min\{D_J : \#J = N - n\},$$

which is $\mathcal{NP}$-hard. To determine a nearly optimal index set J with given cardinality in reasonable time, fast heuristic algorithms of forward and backward type are given in [15] and [13]. The forward strategy is adapted to the special situation of deleting recursively all but one scenarios. The backward strategy consists in the opposite approach, namely, in deleting recursively only one scenario.

Algorithm 1—Forward Reduction:

Step 0: $J^0 := \{1,\ldots,N\}$

Step m+1: $u^{m+1} \in \arg\min_{u \in J^m} D_{J^m \setminus \{u\}}$

$J^{m+1} := J^m \setminus \{u^{m+1}\}$

End: Optimal redistribution (3) w.r.t. $J := J^n$

Algorithm 2—Backward Reduction:

Step 0: $J^0 := \emptyset$

Step m+1: $v^{m+1} \in \arg\min_{v \notin J^m} D_{J^m \cup \{v\}}$

$J^{m+1} := J^m \cup \{v^{m+1}\}$

End: Optimal redistribution (3) w.r.t. $J := J^{N-n}$

C. Scenario tree construction

Next we describe two approaches for constructing scenario trees based on recursive scenario reduction. The first one has been given already in [13]. Both approaches fit into the general tree generation scheme given in [6]. The strategy consists in modifying a given *fan* of individual scenarios by bundling scenarios according to the scenario reduction technique. It can be shown that the constructed trees are much smaller than the given scenario fans, and nevertheless, they are good approximations with respect to the Monge-Kantorovich distance $\hat{\mu}_1$.

Let P be the probability distribution of a fan of multivariate data scenarios $\xi^i = (\xi_1^i, \ldots, \xi_T^i)$ with probabilities π^i, $i = 1,\ldots,N$, i.e., all scenarios coincide at the starting point $t = 1$, i.e., $\xi_1^1 = \ldots = \xi_1^N =: \xi_1^*$. The fan may be regarded as a

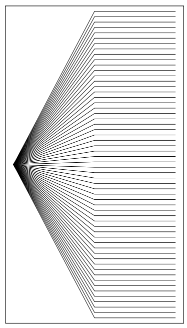

Fig. 1. Scenario fan of individual scenarios.

scenario tree with $1 + N(T-1)$ *nodes*. Given P and $\varepsilon > 0$ we are looking for a probability distribution P_ε such that its scenarios form a scenario tree with root node ξ_1^*, less nodes than P, and

$$\hat{\mu}_1(P, P_\varepsilon) \leq \varepsilon.$$

Again there exist a backward and a forward variant. Let us start with the backward version. It is based on recursive scenario reduction on the time horizon $\{1,\ldots,t\}$, where the time parameter t is reduced recursively from $t = T$ to $t = 2$. For the time horizon $\{1, \ldots, t\}$ we consider the relative costs

$$c_t(\xi, \tilde{\xi}) := \sum_{\tau=1}^{t} \|\xi_\tau - \tilde{\xi}_\tau\|, \tag{5}$$

which corresponds to (2) on $\{1, \ldots, t\}$ for $r = 1$.

Algorithm 3—Backward Construction:

Let $\varepsilon_t > 0$, $t = 2, \ldots, T$, be such that $\sum_{t=2}^{T} \varepsilon_t \leq \varepsilon$.

Step 0: Let $I_{T+1} = \{1, \ldots, N\}$ and $\pi^i_{T+1} = \pi^i$ for all $i = 1, \ldots, N$.

Step m: Set $t = T + 1 - m$.
Determine a scenario index set $I_t \subseteq I_{t+1}$ by scenario reduction, i.e., such that
$\sum_{i \in I_{t+1} \setminus I_t} \pi^i_{t+1} \min_{j \in I_t} c_t(\xi^i, \xi^j) \leq \varepsilon_t$.
Set $\pi^j_t = \pi^j_{t+1} + \sum_{i \in J_{tj}} \pi^i_{t+1}$, where
$J_{tj} = \{i \in I_{t+1} \setminus I_t : j = j_t(i)\}$ and
$j_t(i) \in \arg\min_{j \in I_t} c_t(\xi^i, \xi^j)$, $i \in I_{t+1} \setminus I_t$.

Step T: Construction of P_ε: Determine recursively mappings $\alpha_t : I_T \to I_t$ for $t = T, \ldots, 2$, where $\alpha_T := id|_{I_T}$ and such that
$\alpha_t(i) := \begin{cases} j_t(\alpha_{t+1}(i)), & \alpha_{t+1}(i) \in I_{t+1} \setminus I_t, \\ \alpha_{t+1}(i), & \text{else}, \end{cases}$
for $t = T - 1, \ldots, 2$.
Determine scenarios $\tilde{\xi}^j$ for $j \in I_T$ with $\tilde{\xi}^j_1 = \xi^*_1$, and $\tilde{\xi}^j_t := \xi^{\alpha_t(j)}_t$ for $t = 2, \ldots, T$. Finally, set $\tilde{\pi}^j := \pi^j_T$ and $P_\varepsilon := \sum_{j \in I_T} \tilde{\pi}^j_T \delta_{\tilde{\xi}^j}$.

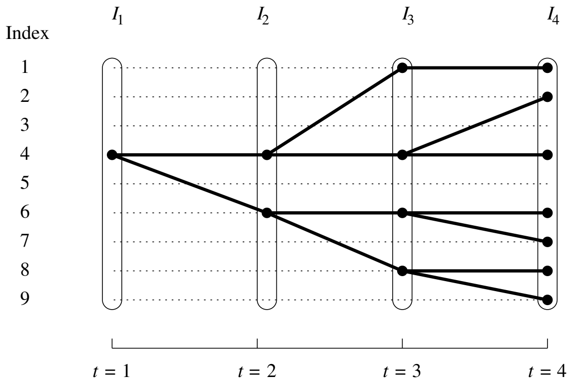

Fig. 2. Illustration of backward scenario tree construction.

It can be shown that $\hat{\mu}_1(P, P_\varepsilon) \leq \varepsilon$ holds for any scenario tree P_ε constructed by Algorithm 3. Figure 3 displays an example demonstrating the recursive reduction of nodes by bundling scenarios for decreasing time horizon.

Next we focus attention on a new forward variant of scenario tree construction. The idea consists in applying the scenario reduction technique repeatedly for increasing time periods from $t = 2$ to $t = T$. The forward method is based on a successive clustering of scenarios, where the number of elements contained in a cluster is recursively reduced. Different from the backward variant, scenario reduction is now applied separately to each cluster, where at time t only ξ_t is taken into account. Instead of (5), let the costs at time t now be defined by

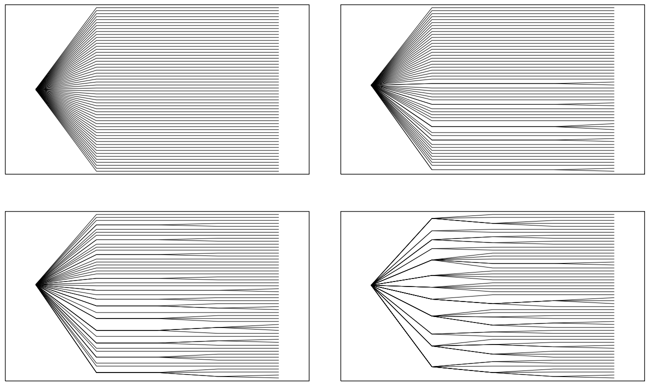

Fig. 3. Example of backward tree construction by reducing nodes of a given scenario fan recursively by modifying the tree structure using scenario reduction and bundling. The time horizon consists of 5 periods.

$$c_t(\xi, \tilde{\xi}) := \|\xi_t - \tilde{\xi}_t\|, \tag{6}$$

which corresponds to (2) at t for $r = 1$.

Algorithm 4—Forward Construction:

Let $\varepsilon_t > 0$, $t = 2, \ldots, T$, be such that $\sum_{t=2}^{T} \varepsilon_t \leq \varepsilon$.

Step 1: Let $I = \{1, \ldots, N\}$ be the first cluster and let $Z_1 := \{I\}$.

Step t: Let $Z_{t-1} = \{I_1, \ldots, I_{l_{t-1}}\}$ the clusters defined in Step $t - 1$.
(1) Chose $\varepsilon_k \geq 0$ such that $\sum_{k=1}^{l_{t-1}} \varepsilon_k \leq \varepsilon_t$
(2) For $k = 1, \ldots, l_{t-1}$: Determine a subset $J_k \subseteq I_k$ and a selection $j_k : I_k \to I_k \setminus J_k$ by scenario reduction such that $j_k(i) = i$ for $i \in I_k \setminus J_k$ and $\sum_{i \in J_k} \pi^i c_t(\xi^i, \xi^{j_k(i)}) \leq \varepsilon_k$.
(3) Set $Z_t = \{j_k^{-1}(i) : i \in I_k \setminus J_k, 1 \leq k \leq l_{t-1}\}$ with $j_k^{-1}(i) := \{j \in I_k : j_k(j) = i\}$.
(4) Define some mapping $\alpha_t : I \to I$ such that $\alpha_t|_{I_k} \equiv j_k$.

Step T+1: Construction of P_ε: Let $Z_T = \{I^T_1, \ldots, I^T_{l_T}\}$ and let $i_1, \ldots, i_{l_T}$ be some indices such that $i_k \in I^T_k$ for $k = 1, \ldots, l_T$.
Determine scenarios $\tilde{\xi}^k$ with $\tilde{\xi}^k_1 = \xi^*_1$ and $\tilde{\xi}^k_t = \xi^{\alpha_t(i_k)}_t$ for $t = 2, \ldots, T$ and $1 \leq k \leq l_T$.
Finally, set $\tilde{\pi}^k := \sum_{i \in I^T_k} \pi^i$ and $P_\varepsilon = \sum_{k=1}^{l_T} \tilde{\pi}^k \delta_{\tilde{\xi}^k}$.

Note that all clusters corresponding to one time step, that are all sets I_k of Z_t, are disjoint and their union cover all indices of $I = \{1, \ldots, N\}$. Hence, the mappings α_t are well defined.

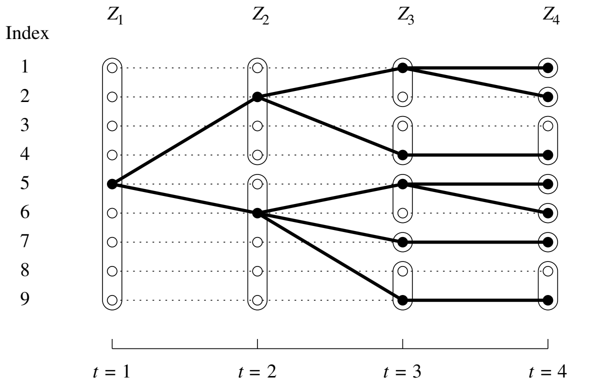

Fig. 4. Illustration of forward scenario tree construction.

Moreover, the values $\alpha_t(i)$ coincide for all indices i contained in the same cluster. See Figure 4 that shows the principle of the algorithm.

For Algorithm 4 the same result can be shown as for the previous Algorithm 3. Namely, the estimate $\hat{\mu}_1(P, P_\varepsilon) \leq \varepsilon$ holds for the probability distribution P_ε of any tree constructed by Algorithm 4. Figure 5 displays an example demonstrating the recursive reduction of nodes by bundling scenarios using the forward method.

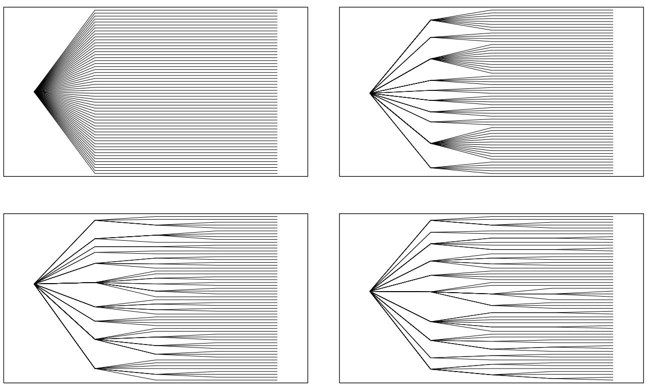

Fig. 5. Example of forward scenario tree construction by reducing nodes of a given scenario fan recursively by modifying the tree structure using scenario reduction and clustering. The time horizon consists of 5 time periods.

III. Application

The scenario tree generation approach was applied to construct scenario trees out of data scenarios provided by Electricité de France (EDF). The data consisted of a finite number of scenarios representing realizations of a bivariate stochastic process whose components are electrical load and water inflow for a time horizon of two years. Since the random data only enter right-hand sides of (in)equality constraints, the relevant probability metric for construction scenario trees is the Monge-Kantorovich distance $\hat{\mu}_1$ (cf. Section II-A).

The time horizon of the data was discretized with three time steps per day, where each time step is associated to a set of daily hours during which the demand does not change much. Table I and II show the discretization of the data for the time horizon of two years and provide the number of scenarios, the total number of time periods and the corresponding number of nodes of the initial scenario fan. The first node (root node) corresponds to the mean value of all scenarios at time period $t = 1$. The weekly amounts of water inflows have been uniformly distributed to the corresponding time steps of the week.

TABLE I
Discretization of the two-year time horizon for the data provided by EDF.

Random variable	Discretization	Number time steps
electrical load	3 per day	2 184
water inflow	weekly	104

TABLE II
Dimension of the initial scenario fan provided by EDF

	Number
scenarios	456
time periods	2 184
initial nodes	995 449

Three series of tests of Algorithms 3 and 4 were performed to generate scenario trees such that

(i) branching is allowed at all time steps,
(ii) branching is only allowed at the beginning of a day,
(iii) branching is only allowed at the beginning of a week.

To measure the distances between the original and approximate probability distributions the relative tolerance $\varepsilon_{rel} := \frac{\varepsilon}{\varepsilon_{max}}$ was used in all test runs, where ε_{max} is the best possible distance between the probability distribution of the initial scenario fan and the distribution of one of its scenarios endowed with unit mass.

All test runs were performed on a PC with a 3 GHz Intel Pentium CPU and 1 GByte main memory.

A. *Results of backward construction*

For the backward variant of scenario tree construction individual tolerances ε_t at branching points were chosen recursively such that

$$\begin{aligned} \varepsilon_T &= \varepsilon \cdot (1-q), \quad q \in (0,1) \quad \text{and} \\ \varepsilon_t &= q \cdot \varepsilon_{t+1}, \quad t = T-1, \ldots, 2. \end{aligned}$$

According to our numerical experience a choice of $q \in (0, 1)$ closer to 1 leads to a higher number of remaining scenarios and branching points (stages). Choosing q closer to 0 leads to the opposite effect. For the test runs of Algorithm 3 we used $q = 0.95$.

Tables III–V display the numerical results for the series of tests (i)–(iii) and different relative tolerances. The second and third column compare the sizes of the initial scenario fan and the constructed scenario tree in terms of the numbers of scenarios and nodes, respectively. The last but one column contains the number of stages, i.e., the number of time periods where branching occurs. The computing times for constructing the trees can be found in the last column. The computing time already contains the CPU time of (about) 100 seconds for computing the distances of scenarios which are needed in all test runs.

TABLE III
RESULTS FOR BACKWARD TREE CONSTRUCTION WITHOUT BRANCHING RESTRICTION

ε_{rel}	Scenarios	Nodes	Stages	Time (sec)
0.10	442	584 270	151	172.86
0.20	429	371 046	150	129.11
0.30	417	268 201	146	117.42
0.40	405	193 014	135	110.83
0.50	393	140 536	115	106.30

TABLE IV
RESULTS FOR BACKWARD TREE CONSTRUCTION WITH DAILY BRANCHING RESTRICTION

ε_{rel}	Scenarios	Nodes	Stages	Time (sec)
0.10	442	584 793	128	134.17
0.20	429	373 569	124	115.47
0.30	417	269 850	125	110.41
0.40	405	196 182	120	107.4
0.50	393	144 009	110	104.93

TABLE V
RESULTS FOR BACKWARD TREE CONSTRUCTION WITH WEEKLY BRANCHING RESTRICTION

ε_{rel}	Scenarios	Nodes	Stages	Time (sec)
0.10	442	589 575	88	118.47
0.20	429	397 047	83	110.65
0.30	416	293 403	86	108.40
0.40	405	219 714	83	106.15
0.50	393	170 520	81	105.16

It turns out that for a small relative tolerance an approximate scenario tree can be constructed that contains only 50% of the original nodes. The pictures of Figure 6 and 7 show the structure of two generated scenario trees with weekly branching structure and epsilon tolerances $\varepsilon_{rel} = 0.2$ and $\varepsilon_{rel} = 0.5$, respectively.

B. Results of forward construction

In a second series of tests scenario trees were constructed out of the EDF data by the new Algorithm 4. In case of forward tree construction individual tolerances ε_t at branching points were chosen such that

$$\varepsilon_t \quad = \quad \frac{\varepsilon}{T}\left[1+\overline{q}\left(1-\frac{t}{T}\right)\right], \quad t=2,\ldots,T,$$

where $\overline{q} \in [0,1]$ is a parameter, that affects the branching structure of the constructed trees very similar to the case of backward construction. For the test runs we used $\overline{q} = 1$.

Tables VI-VIII provide numerical results for Algorithm 4. Just as before, the tables correspond to the series of tests (i)-(iii), i.e., the first one contains results for trees without branching restriction, the second for trees with a daily branching structure, and the last by allowing branching only at the beginning of a week.

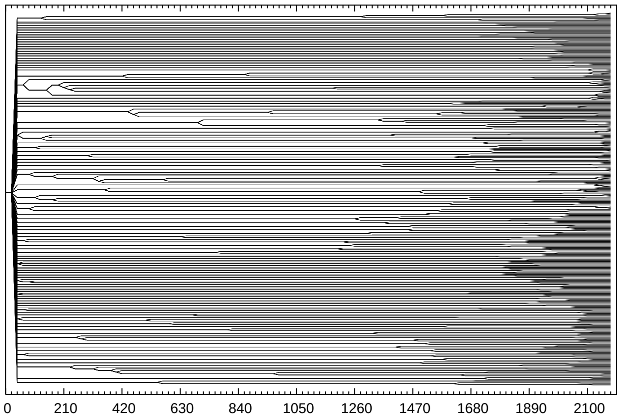

Fig. 6. Generated scenario tree based on EDF-data obtained by the backward construction with $\varepsilon_{rel} = 0.2$ and weekly branching structure.

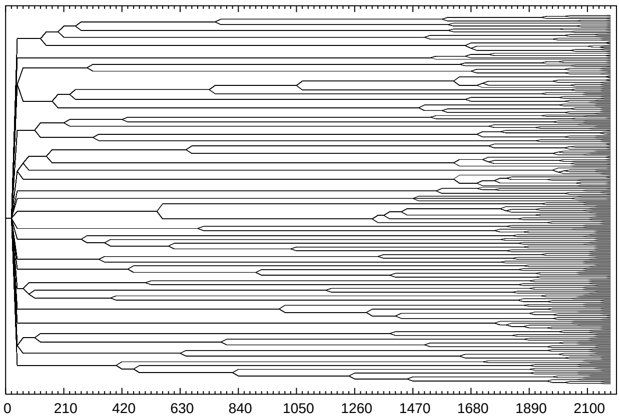

Fig. 7. Generated scenario tree based on EDF-data obtained by the backward construction with $\varepsilon_{rel} = 0.5$ and weekly branching structure.

The numerical results illustrate that the forward variant of scenario tree construction performs as well as the backward version. Nevertheless, there are certain differences. Namely, it turns out that, for small relative tolerances, the trees contain less nodes in case of the backward tree construction compared to the forward variant. For increasing relative tolerances the new forward construction algorithm provides trees containing less nodes than the backward counterpart.

Figure 8 and 9 illustrate the generated scenario trees with weekly branching structure for $\varepsilon_{rel} = 0.4$ and $\varepsilon_{rel} = 0.5$. For these trees it turns out that about 15% of all nodes are sufficient to guarantee 40% accuracy, while 6% of the nodes still guarantee 50% accuracy.

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TABLE VI

RESULTS FOR FORWARD TREE CONSTRUCTION WITHOUT BRANCHING RESTRICTION

ε_{rel}	Scenarios	Nodes	Stages	Time (sec)
0.10	378	743 087	129	108.11
0.20	305	529 994	162	109.15
0.30	216	289 324	161	114.18
0.40	145	138 175	121	134.11
0.50	93	67 696	84	202.42

TABLE VII

RESULTS FOR FORWARD TREE CONSTRUCTION WITH DAILY BRANCHING RESTRICTION

ε_{rel}	Scenarios	Nodes	Stages	Time (sec)
0.10	380	739 545	101	106.72
0.20	309	521 871	131	107.33
0.30	217	299 520	137	108.99
0.40	144	139 236	108	115.95
0.50	92	64 569	74	149.43

TABLE VIII

RESULTS FOR FORWARD TREE CONSTRUCTION WITH WEEKLY BRANCHING RESTRICTION

ε_{rel}	Scenarios	Nodes	Stages	Time (sec)
0.10	389	746 613	49	106.53
0.20	300	509 103	57	106.84
0.30	228	310 653	64	107.59
0.40	163	151 809	69	109.78
0.50	92	60 501	46	119.12

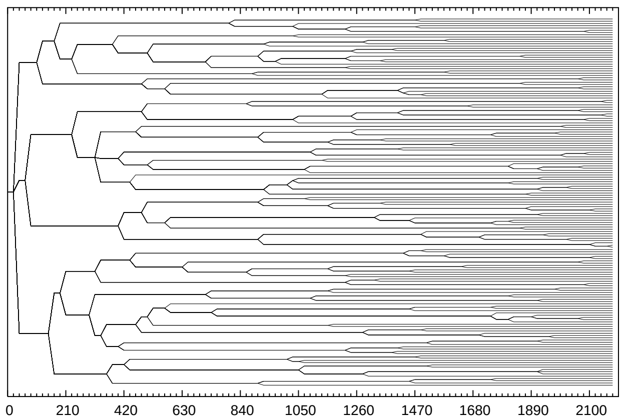

Fig. 8. Generated scenario tree based on EDF-data obtained by the forward construction with $\varepsilon_{rel} = 0.4$ and weekly branching structure.

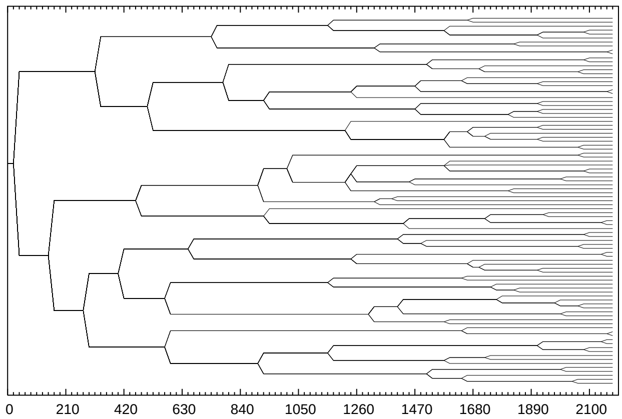

Fig. 9. Generated scenario tree based on EDF-data obtained by the forward construction with $\varepsilon_{rel} = 0.5$ and weekly branching structure.

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