# An Introduction to Mesh Generation Methods and Softwares for Scientific Computing

## Part I

TU Berlin

Nov. 24, 2016

- My name is Hang Si. I'm a senior researcher in Weierstrass Institute (WIAS) in Berlin.

- My main research interest is mesh generation for scientific computing. I'm developing  the software, TetGen -- a Delaunay-based tetrahedral mesh generator. It is freely available for academic use at http://www.tetgen.org.

- Homepage: http://www.wias-berlin.de/people/si

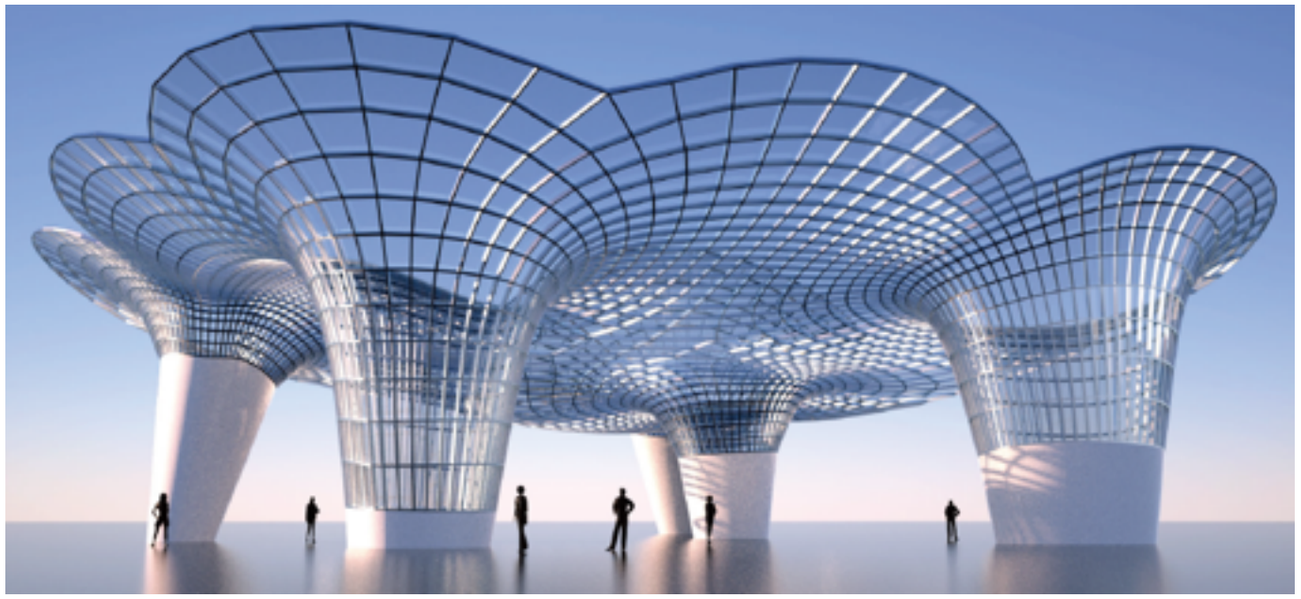
The topic of this lecture is about **mesh generation**.

But first, what are **meshes**?

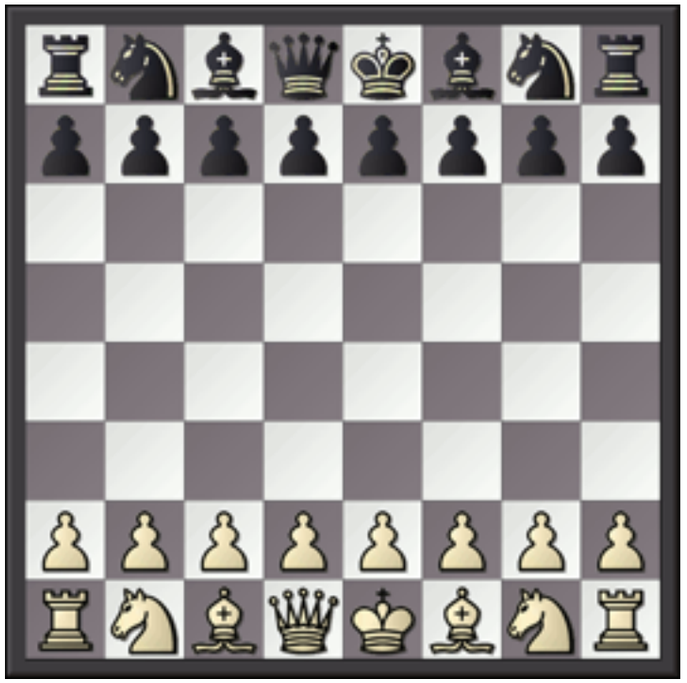
a view from the airplane

the wing of a dragonfly

# Meshes in Nature Objects



# Mdeshes in Architechures

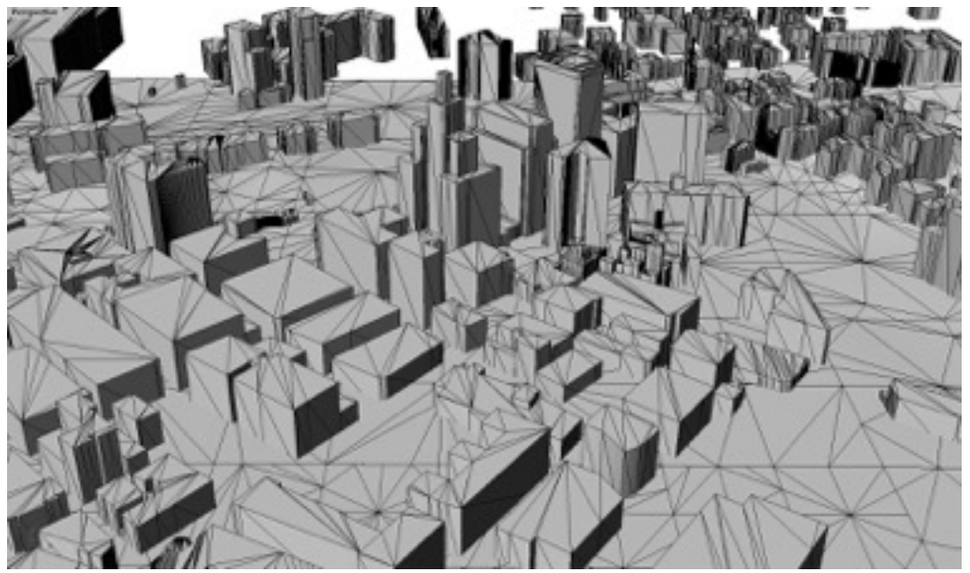


# Meshes in Daily life

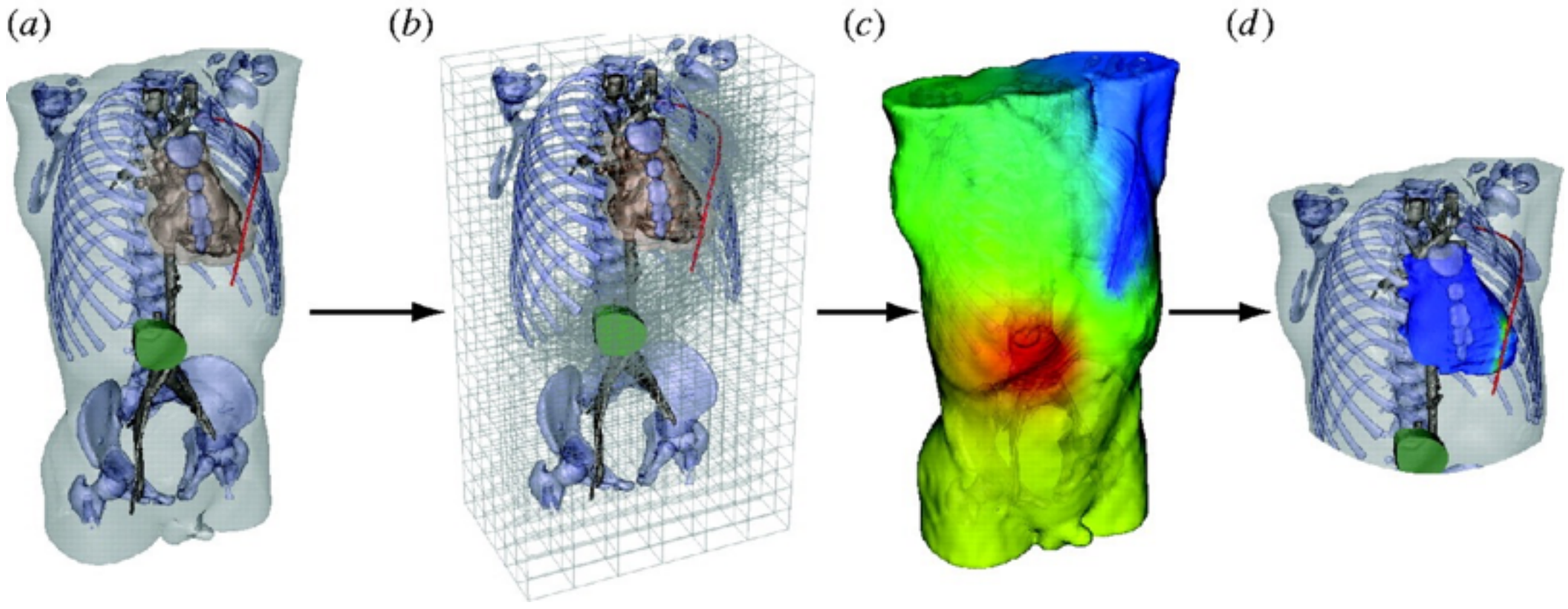
- **Meshes** are partitions of geometric objects.
- **Meshes** are discrete representations of continuous objects.
- **Meshes** may be called differently in different areas and literatures, like **grids**, **triangulations**, etc.

# Geo Information Science (GIS)

Google earth

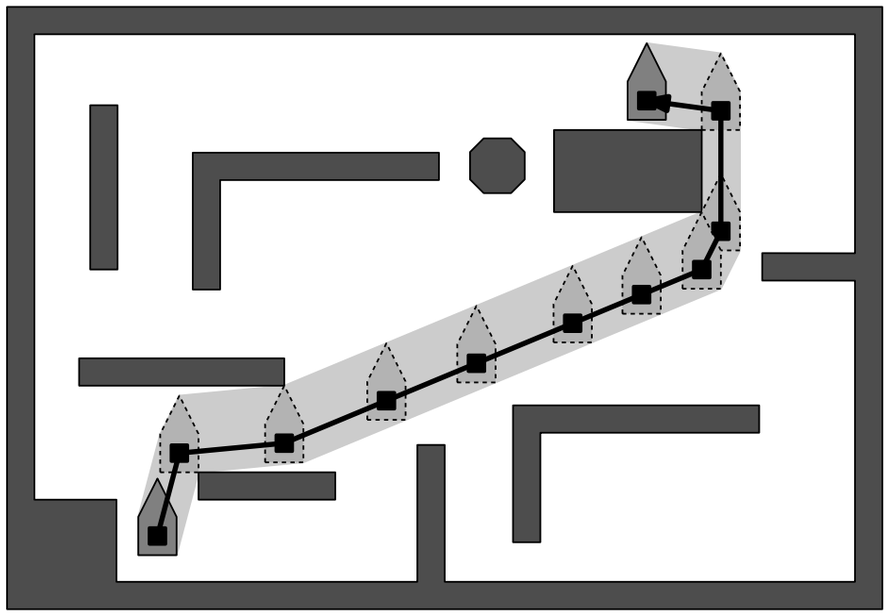


# Scientific Visualization



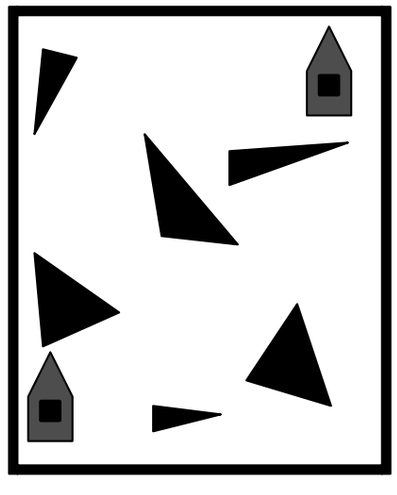
# **Meshes** Are Tools to Solve Complicated Problems

# Robot Route Plan

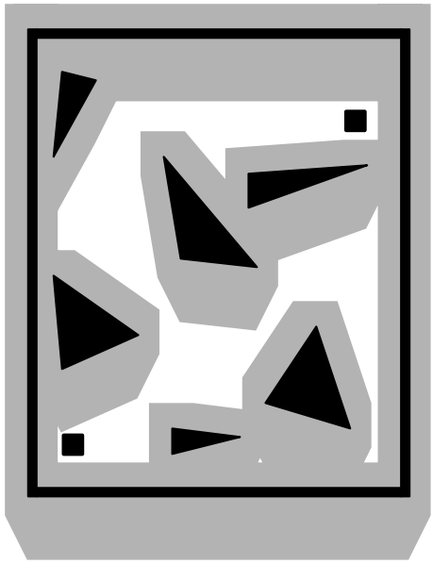


Voronoi diagram with obstacles

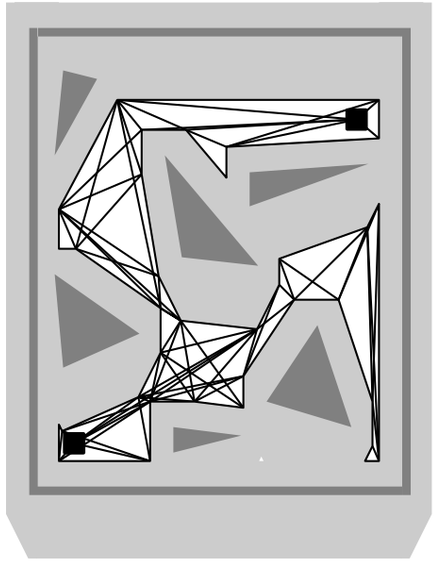
work space



configuration space

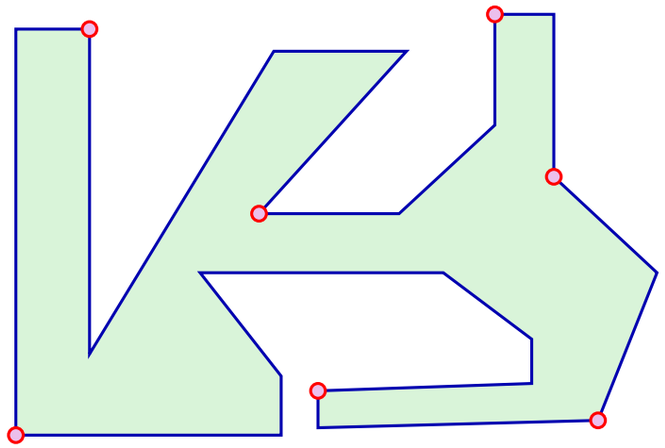


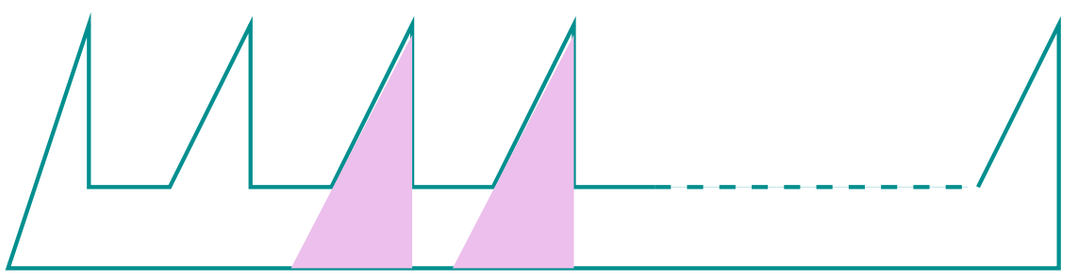
visibility graph

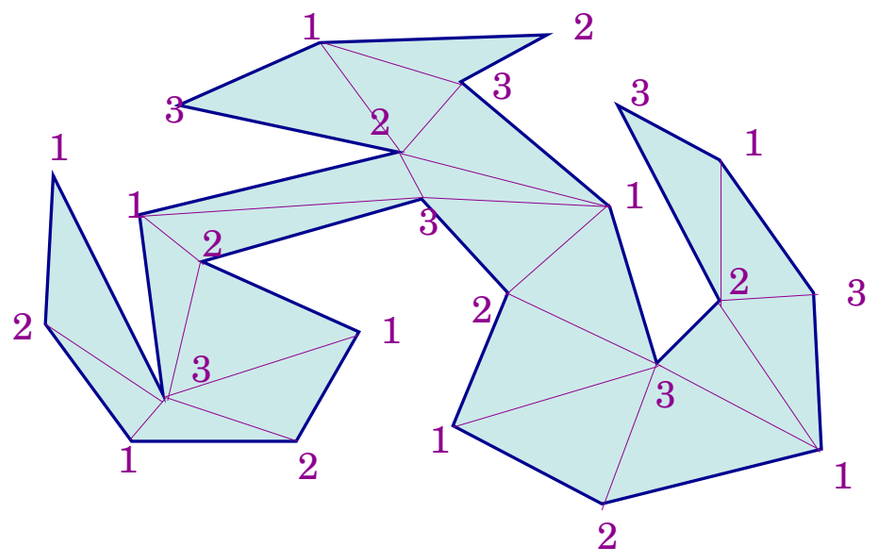


# Example: The Art Gallery Problem

How many cameras are needed to guard a museum?



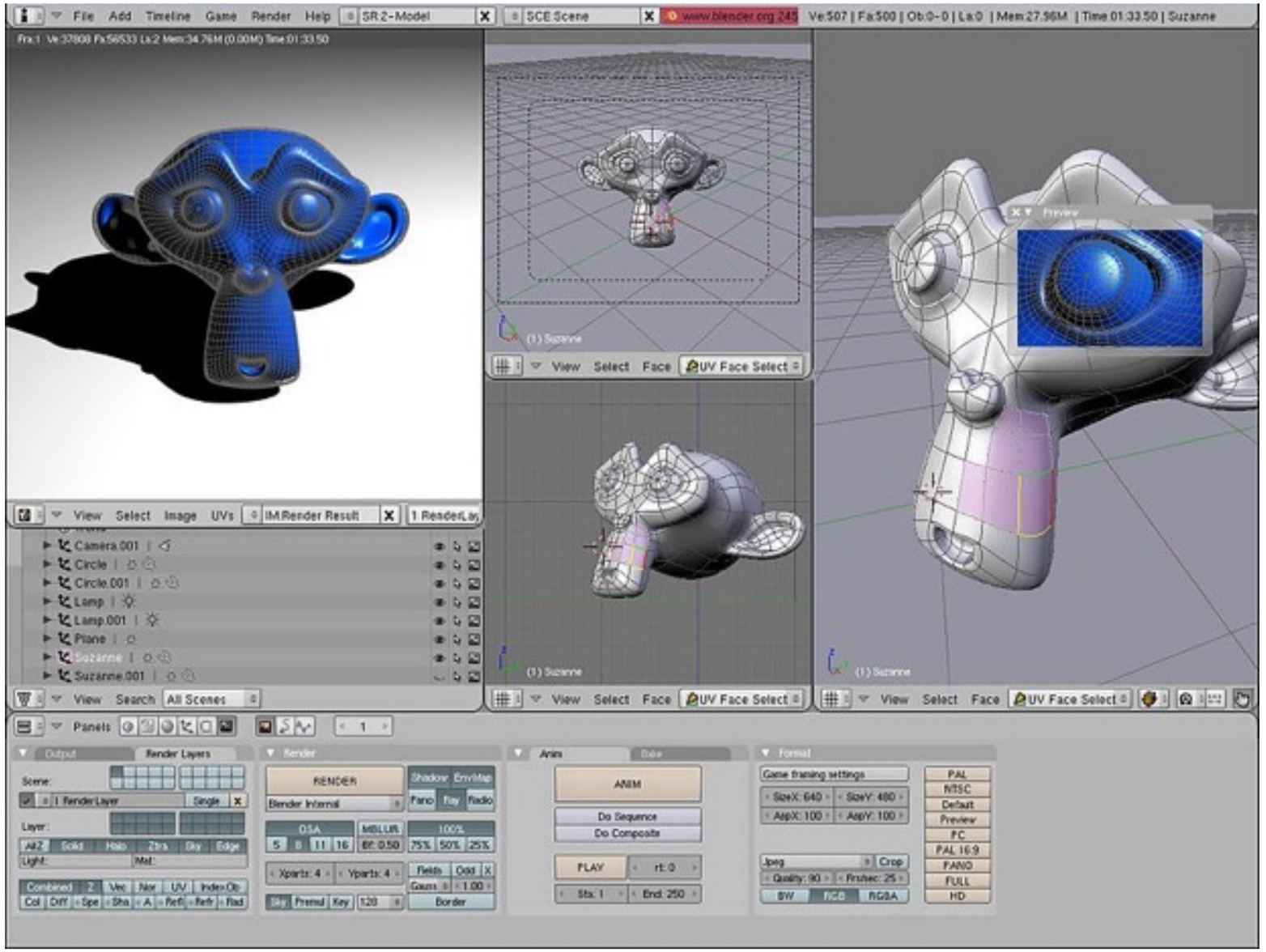





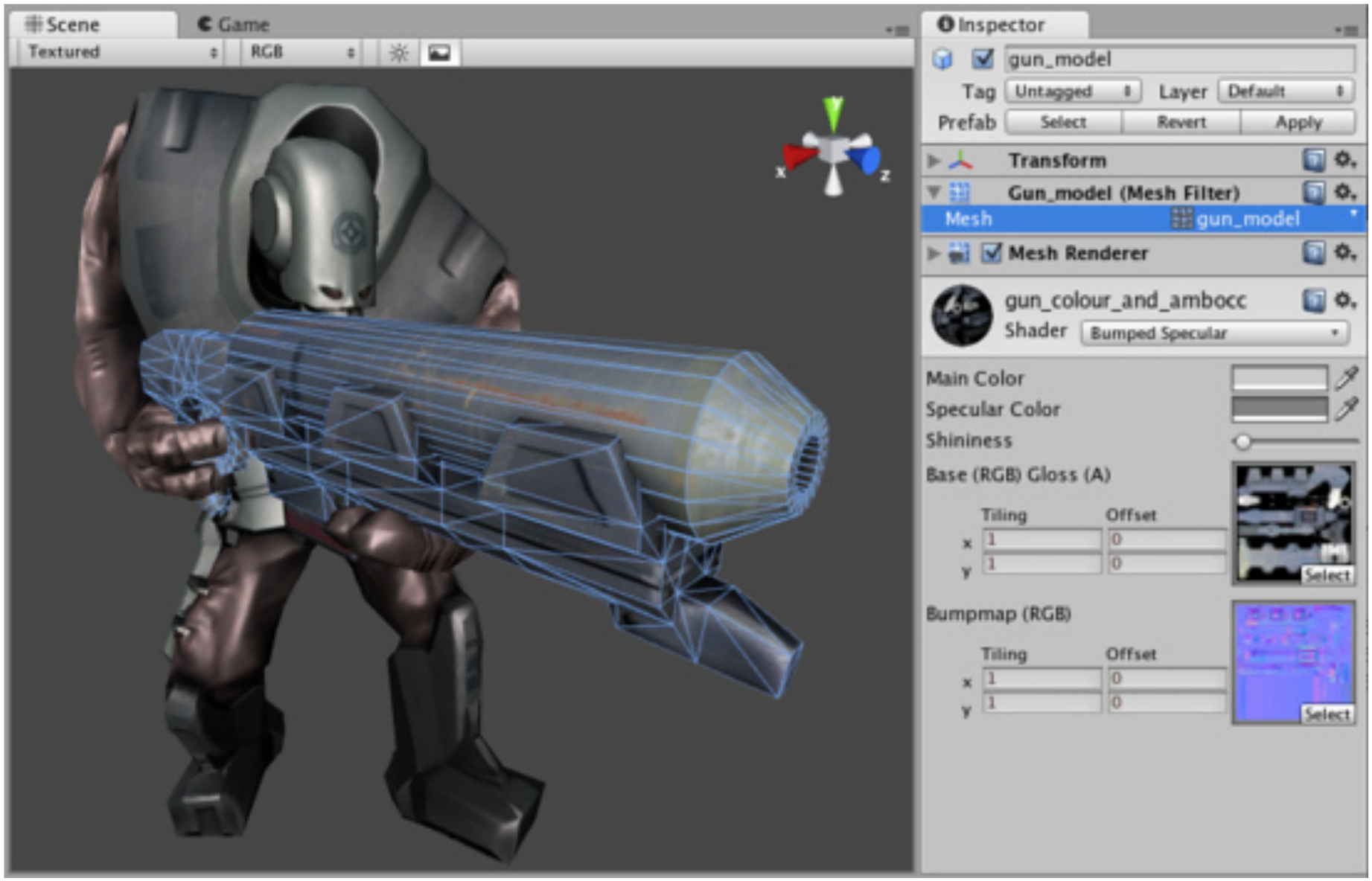

- **Triangulate $P$. 3-color it.**

# **Meshes** are Backbones of 3D Computations and Applications

# Solid & Geometric Modeling

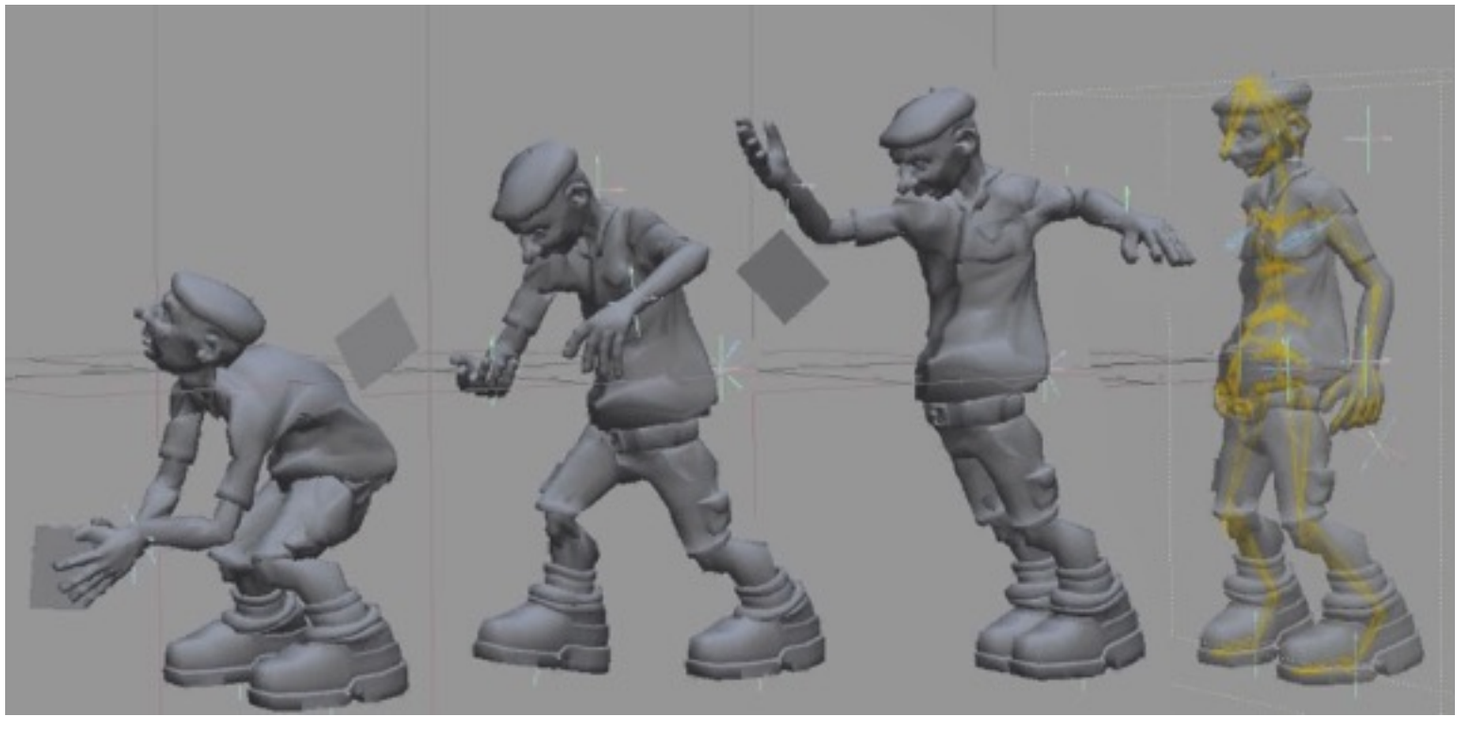



Blender

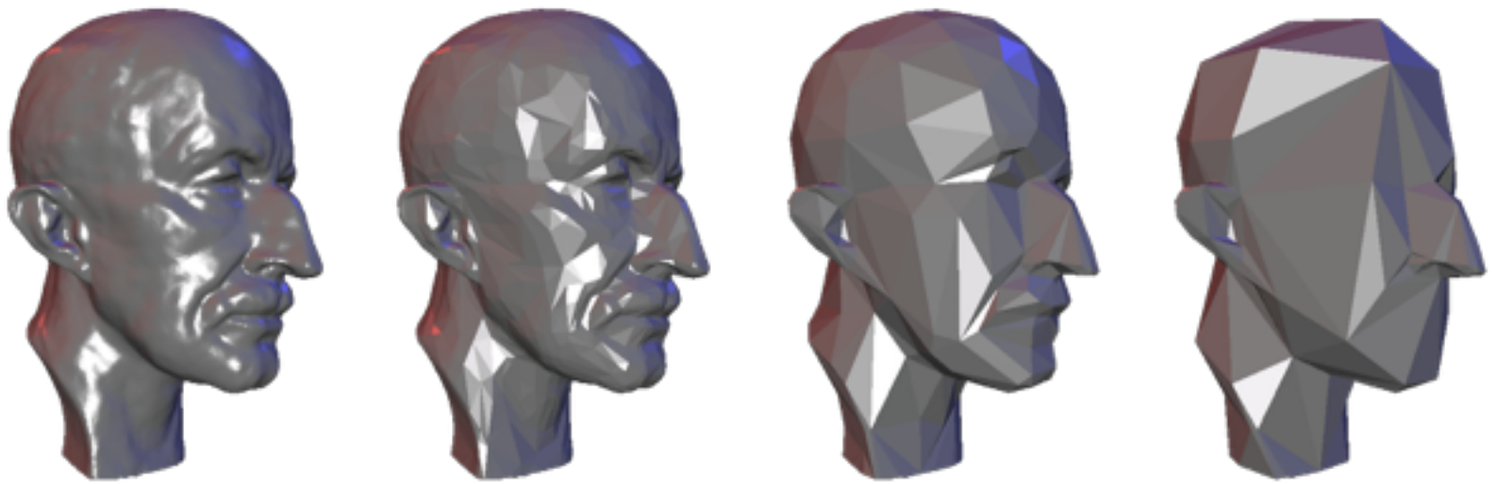



# Computer Games

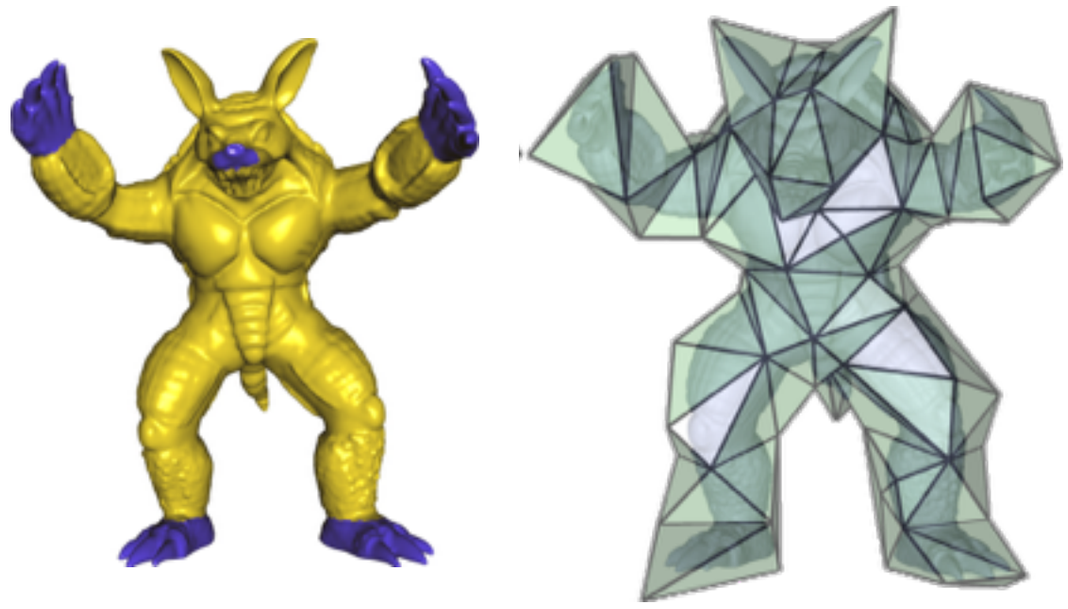
# Computer Animations



# Geometry Processing



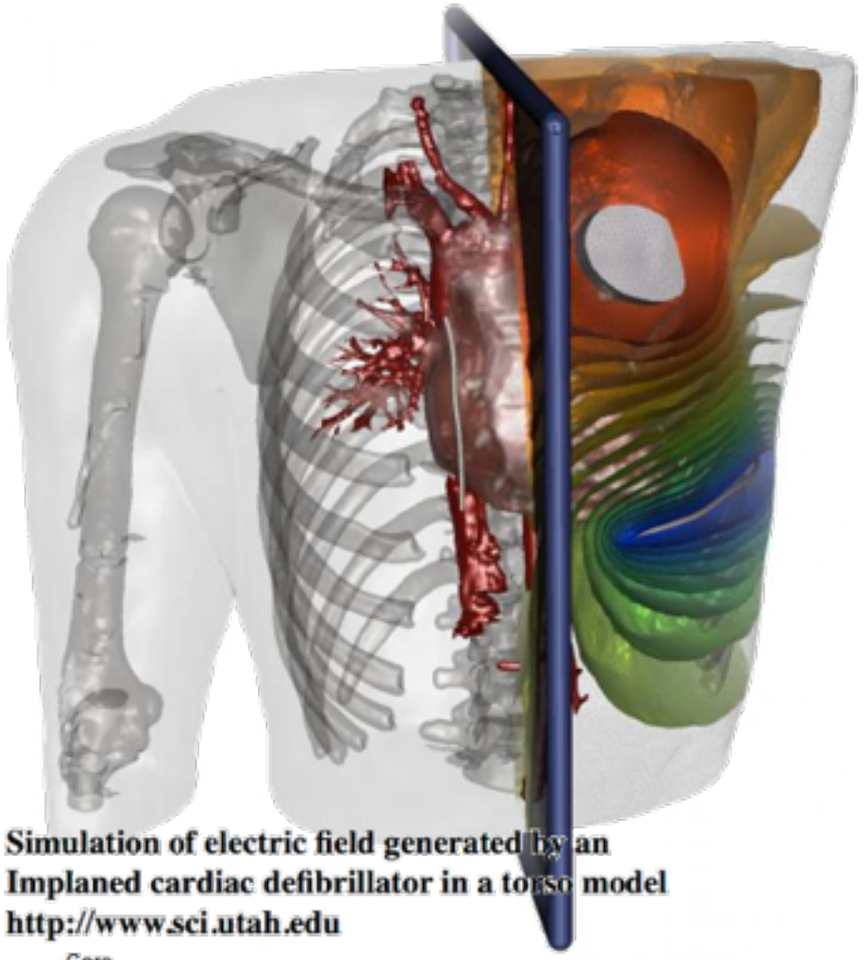
Multiresolution of scanned data (P. Alliez)



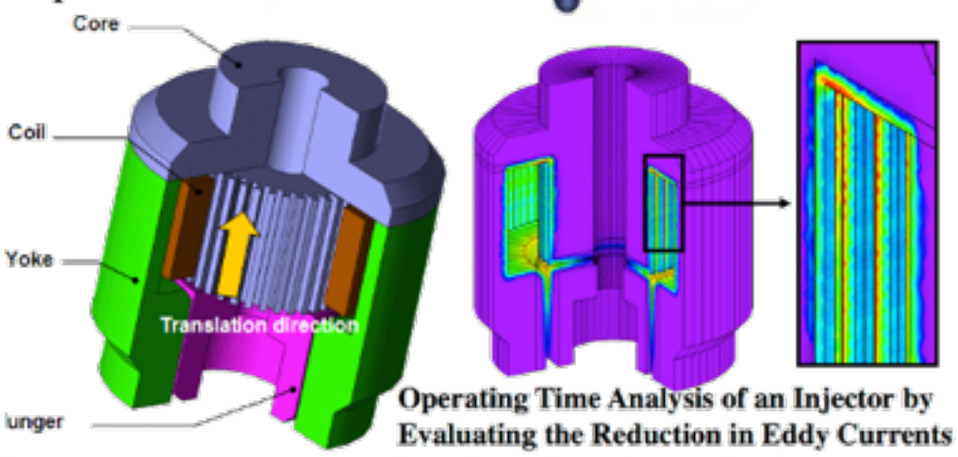
Skinning of 3D Objects (A. Jacobson)

Hobbit 3

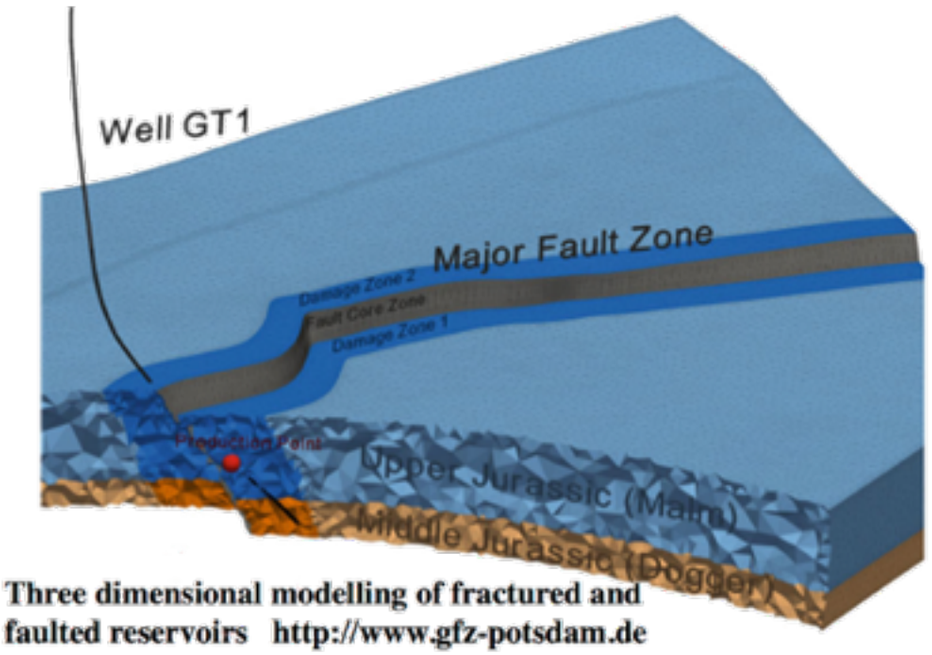
# Numerical Simulation



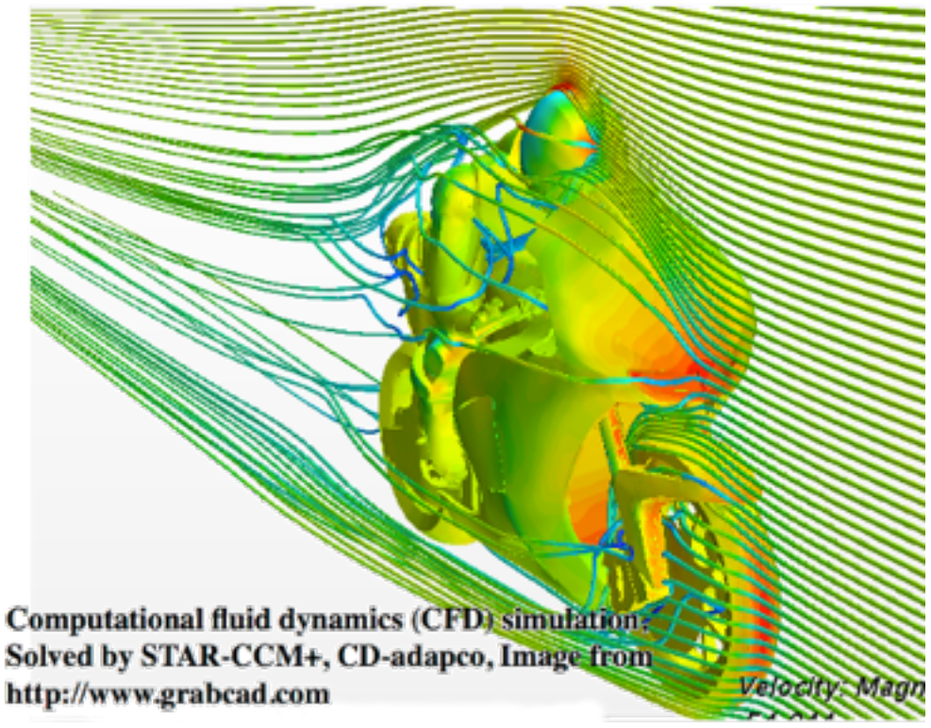
Simulation of electric field generated by an
Implaned cardiac defibrillator in a torso model
http://www.sci.utah.edu




Operating Time Analysis of an Injector by
Evaluating the Reduction in Eddy Currents
http://www.jmag-international.com

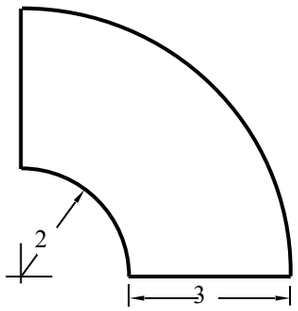



Three dimensional modelling of fractured and
faulted reservoirs http://www.gfz-potsdam.de

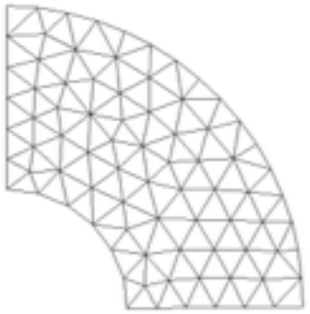



Computational fluid dynamics (CFD) simulation:
Solved by STAR-CCM+, CD-adapco, Image from
http://www.grabcad.com

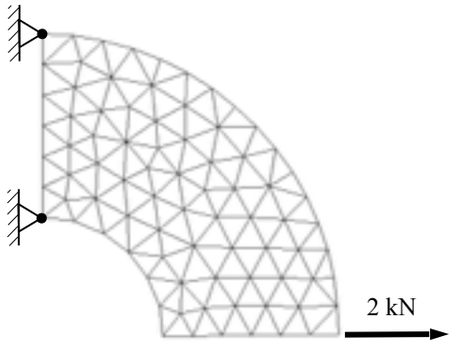
# Simulation Process




1. Build CAD Model

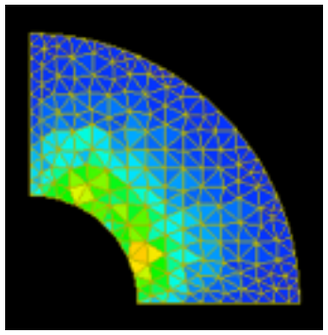


2. Mesh



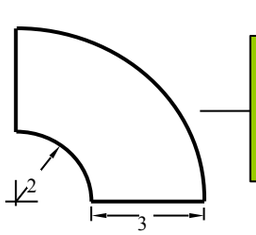

3. Apply Loads and Boundary Conditions

4. Computational Analysis



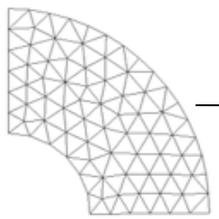
5. Visualization

# Adaptive Simulation Process

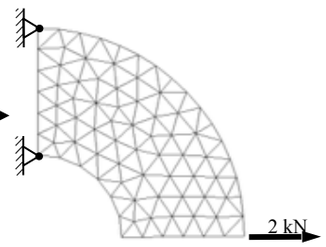



1. Build CAD Model

User supplies meshing parameters



2. Mesh




3. Apply Loads and Boundary Conditions

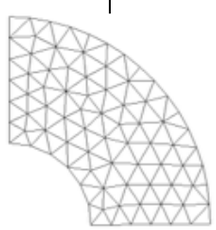
Adaptivity Loop

4. Computational Analysis


5. Error Estimation

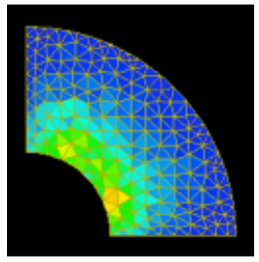
Error > ε

Analysis Code supplies meshing parameters

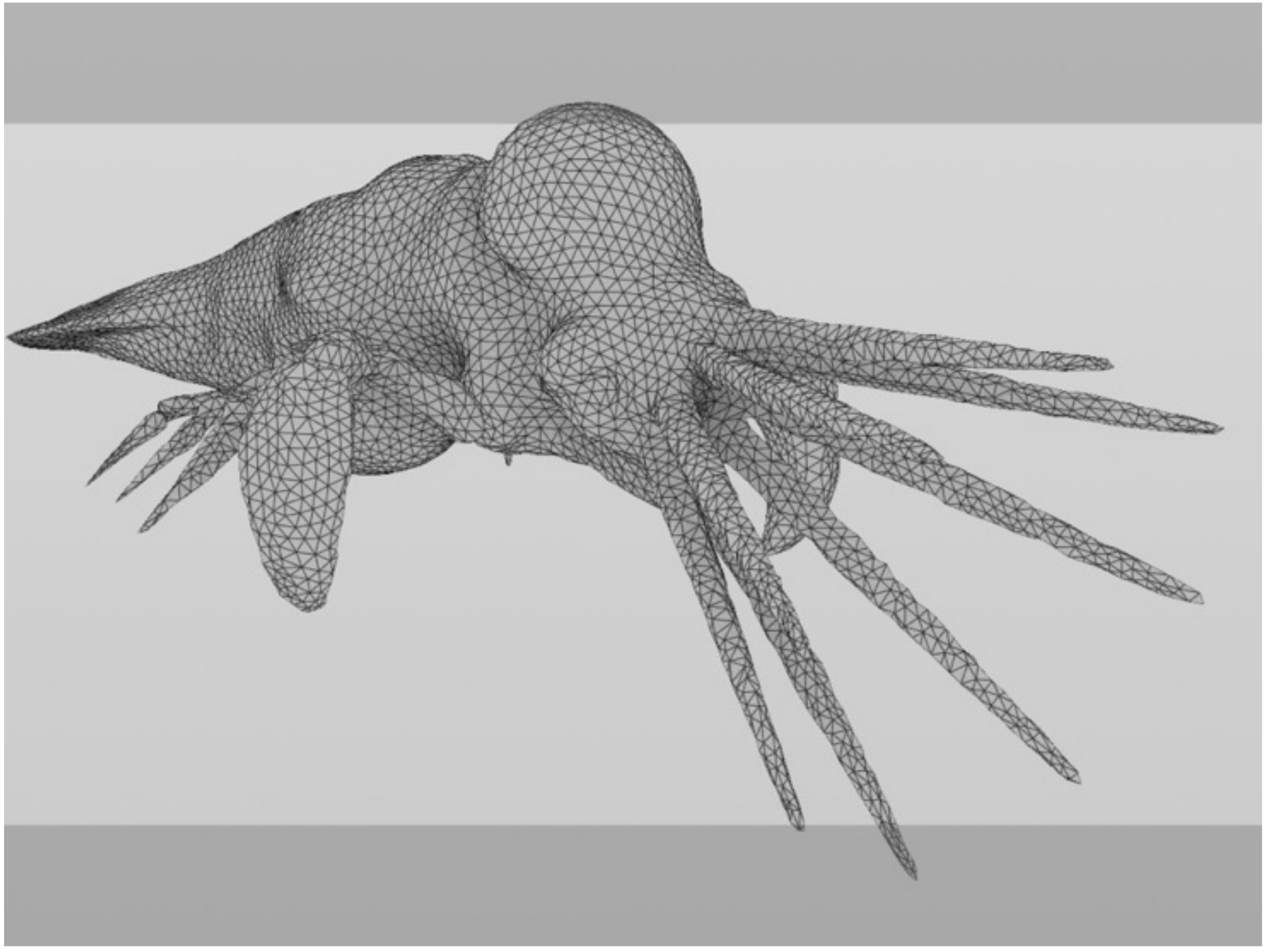


6. Remesh/Refine/Improve

Error < ε



7. Visualization



Houdini

# What is Mesh Generation?

- **Mesh generation** is a practice of generating a polygon or polyhedral mesh that approximates a geometric domain.

  — Wikipedia

# Why Studying Mesh Generation?

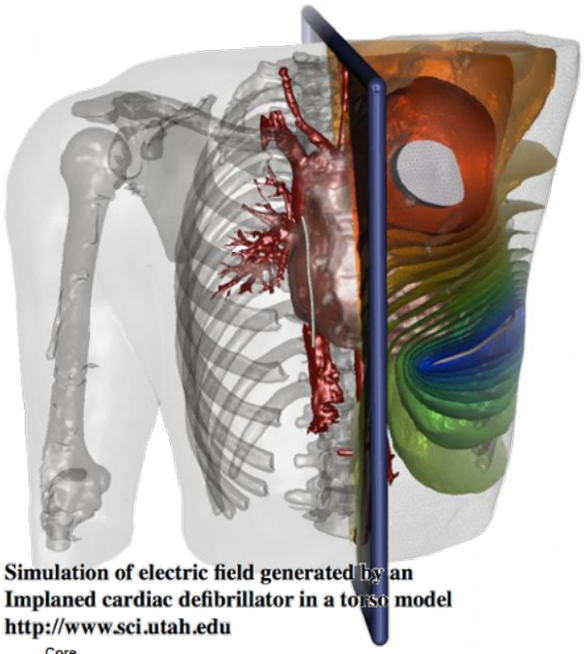
Mesh Generation is the bottleneck of applications:
No mesh, No Run!

# However, the importance of mesh generation is often ignored!

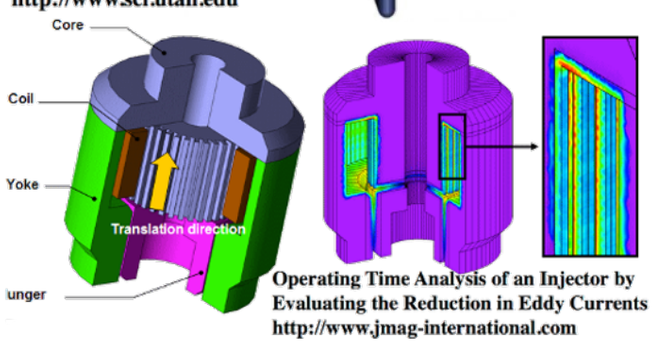
Typical text which appear in books and literatures:

- … let's assume there is a mesh …
- … let T_h be a triangulation whose size h tends to 0, …

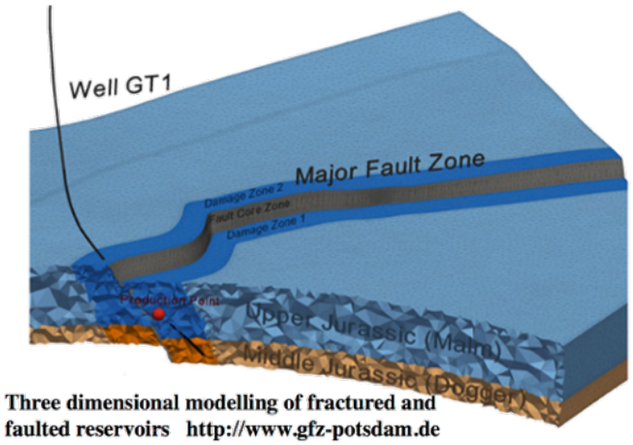
- Automatically generating meshes from arbitrary 3d geometries is very challenging.



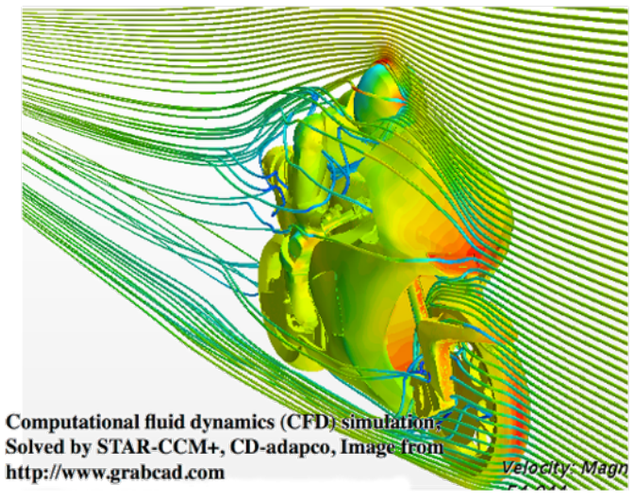
Simulation of electric field generated by an Implaned cardiac defibrillator in a torso model
http://www.sci.utah.edu



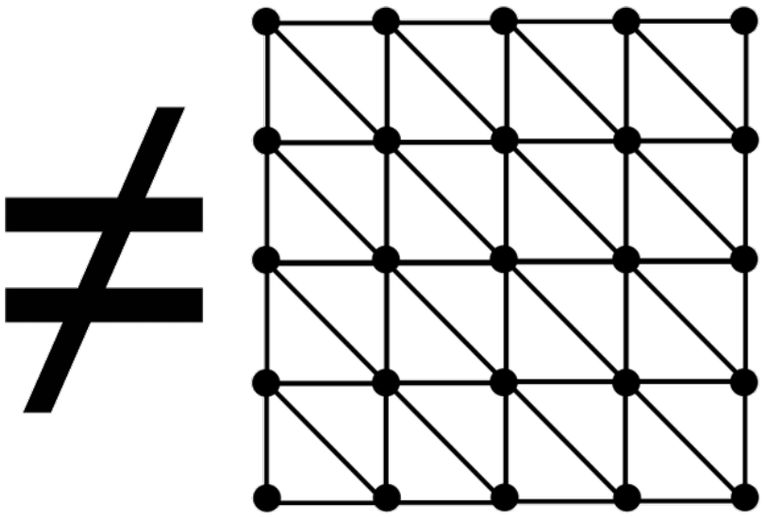

Operating Time Analysis of an Injector by Evaluating the Reduction in Eddy Currents
http://www.jmag-international.com



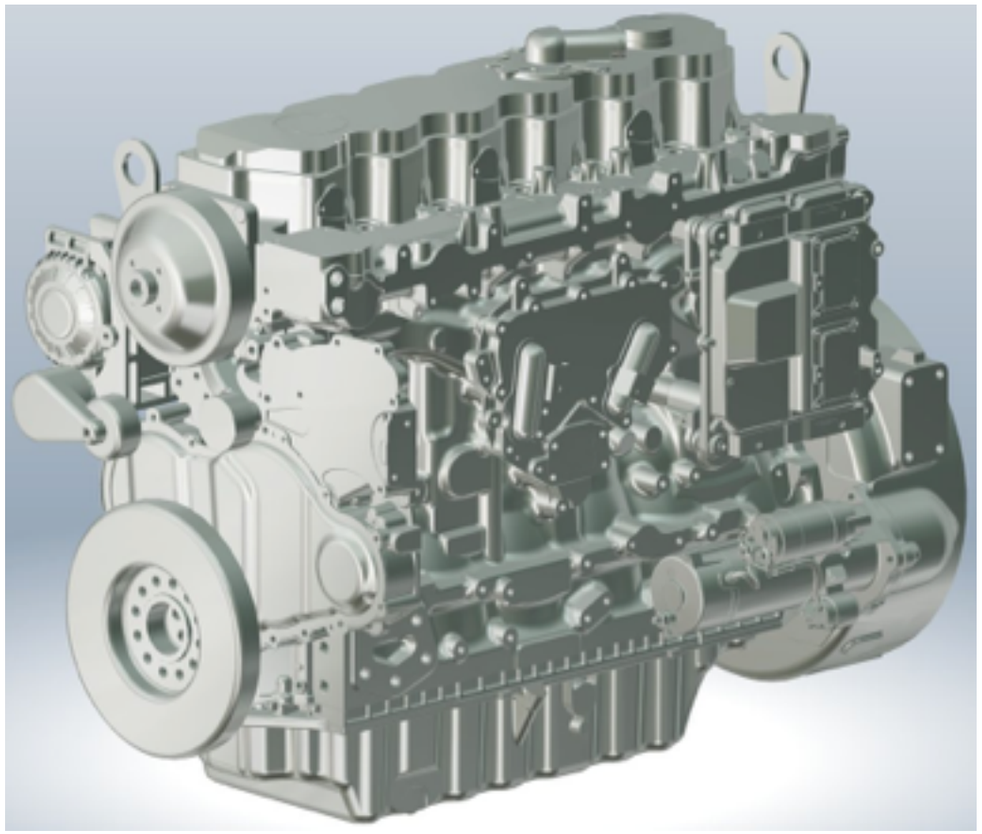

Three dimensional modelling of fractured and faulted reservoirs http://www.gfz-potsdam.de

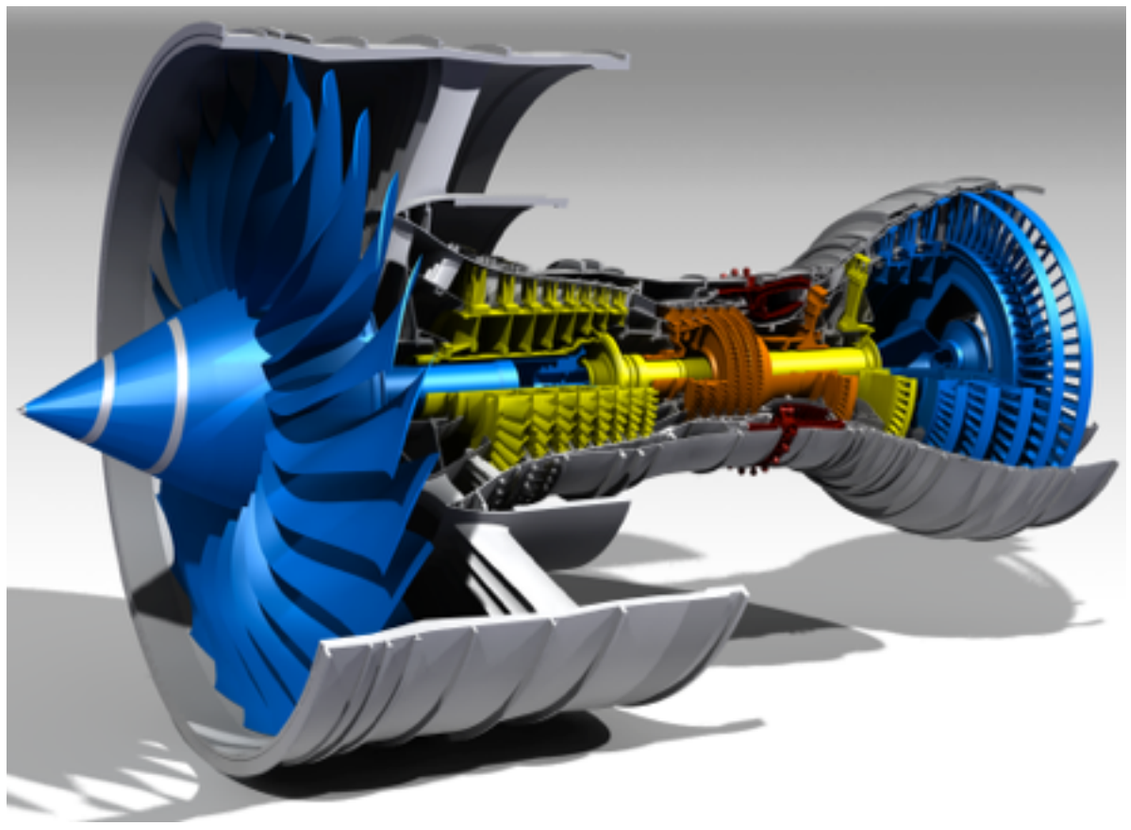



Computational fluid dynamics (CFD) simulation. Solved by STAR-CCM+, CD-adapco, Image from http://www.grabcad.com



- Mesh generation can take orders of magnitude more man-hours — **J. Thompson** (Prof. of Areospace, Pioneer of mesh generation techniques)

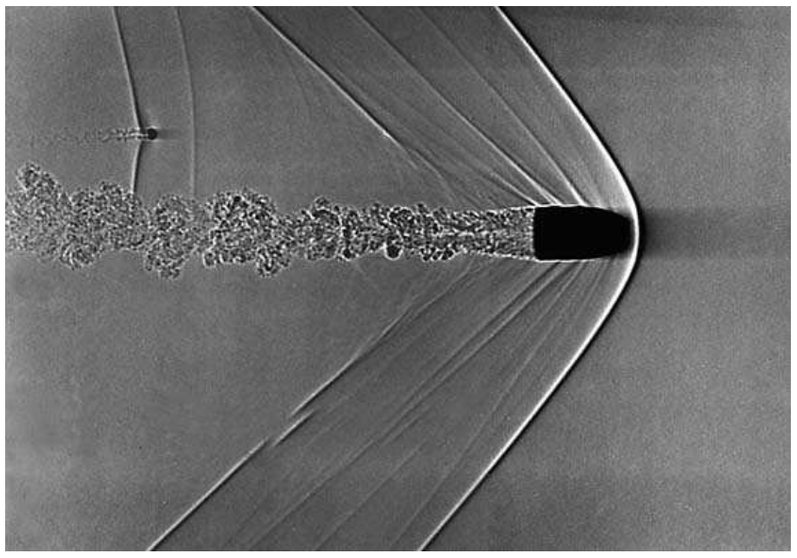


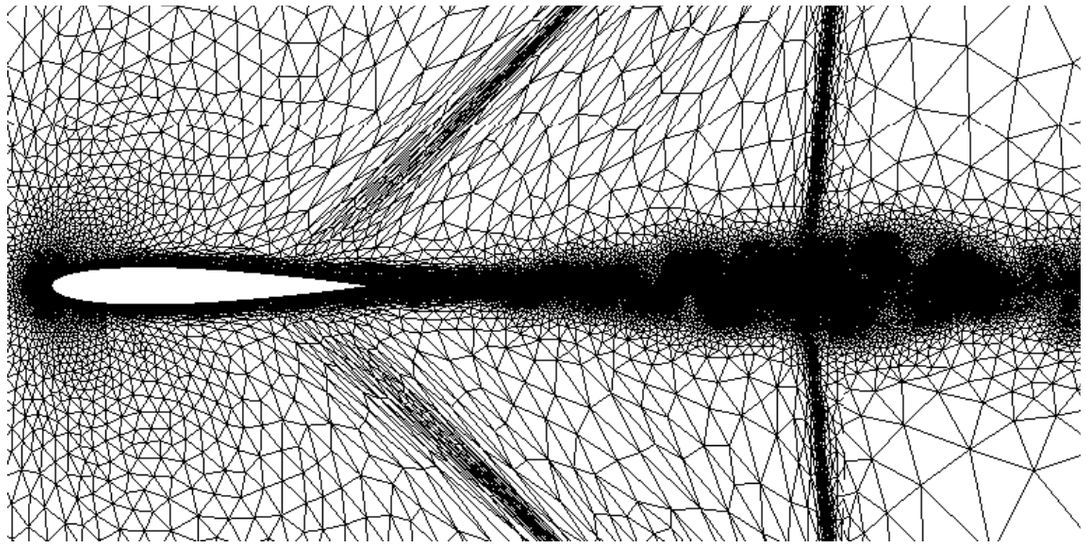


Some CAD models, freely available from http://www.grabcad.com

- Numerical simulation requires high quality meshes for achieving accuracy and efficiency.

- To generate a ``good quality" mesh for the underlying physical problem is a very challenging task.

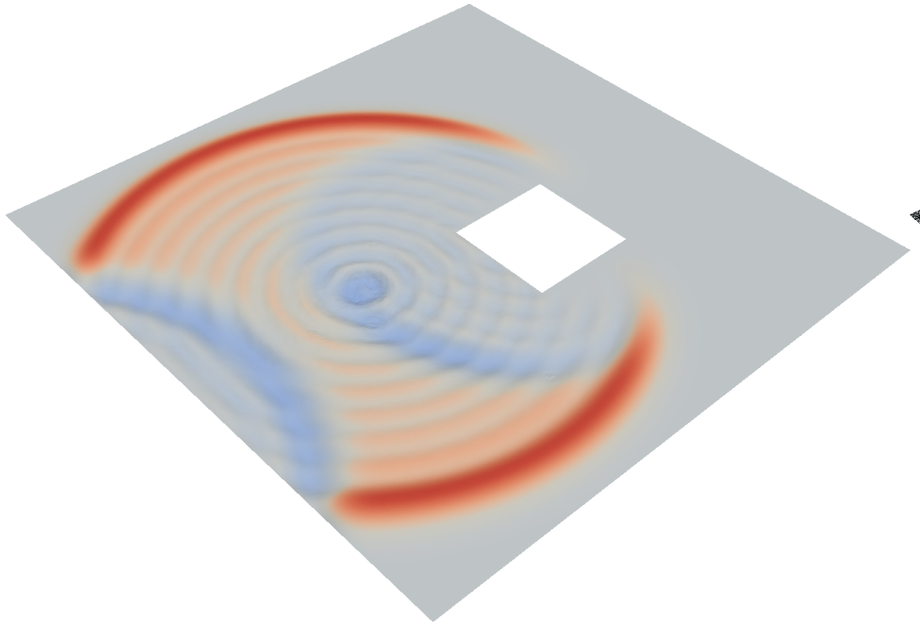


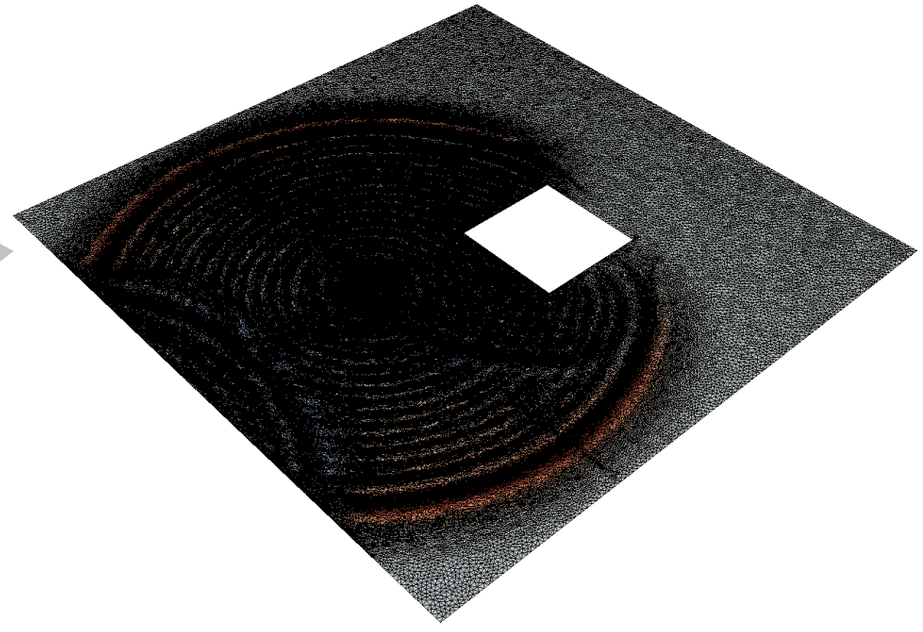


## Example: Adaptive FEM for the Wave Equation

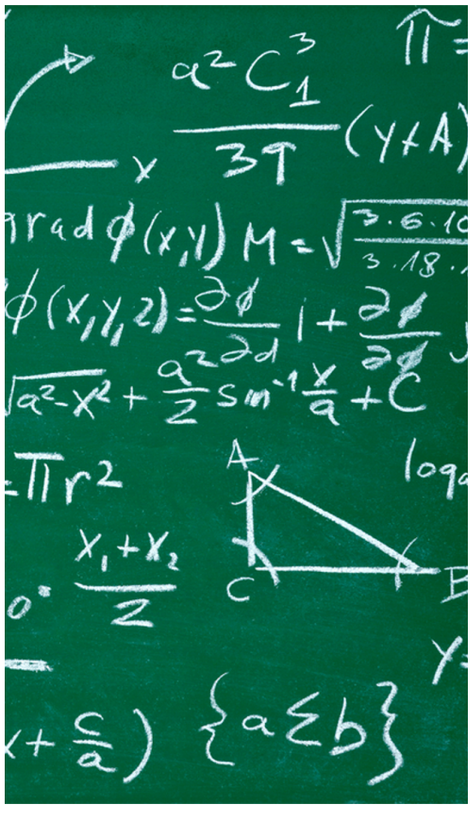
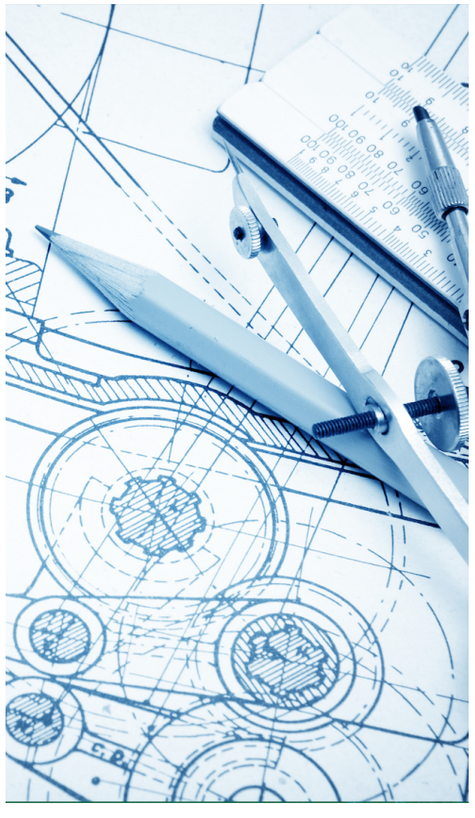
$$\begin{cases} \frac{\partial^2 u}{\partial t^2} - \mu \Delta u &= f \qquad \text{in } \Omega\,, \\ u &= 0 \qquad \text{in } \partial\Omega\,, \end{cases}$$

here $\mu = 1.$, $f$ discrete Dirac function.





- Mesh generation is a topic in which a meaningful combination of different approaches to problem solving is inevitable — **H. Edelsbrunner** (Prof. of Math & Computer Science, Pioneer of computational geometry and topology)

| Math | Engineering | Computer Science | Art |
|---|---|---|---|
|  |  |  |  |

# Resources for Studying Mesh Generations

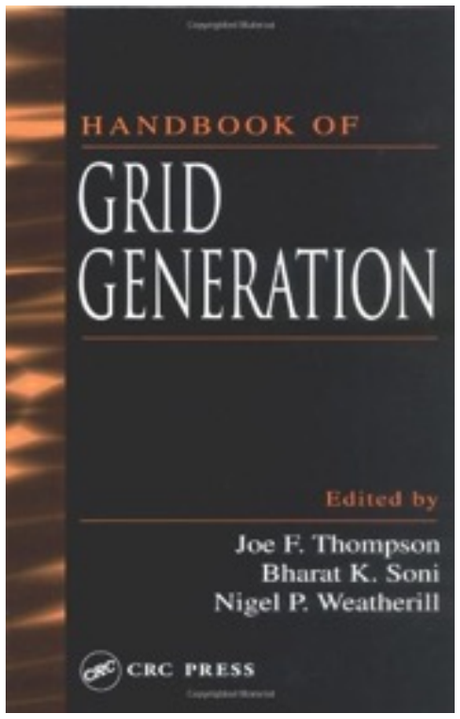
# Resources on the web

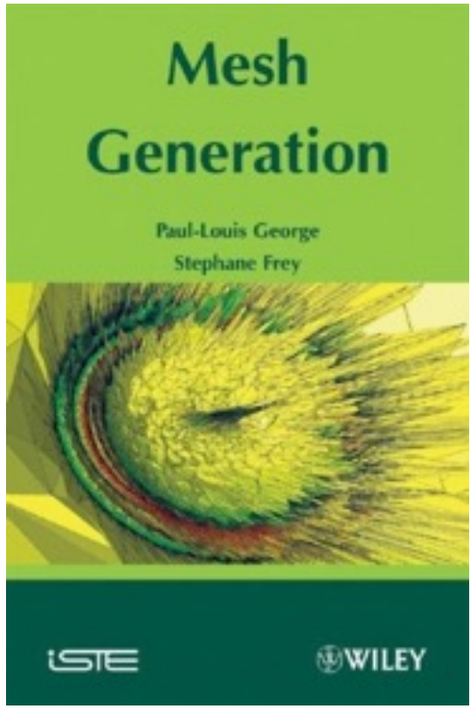
- S. Owen, survey of mesh generation techniques, 1998.

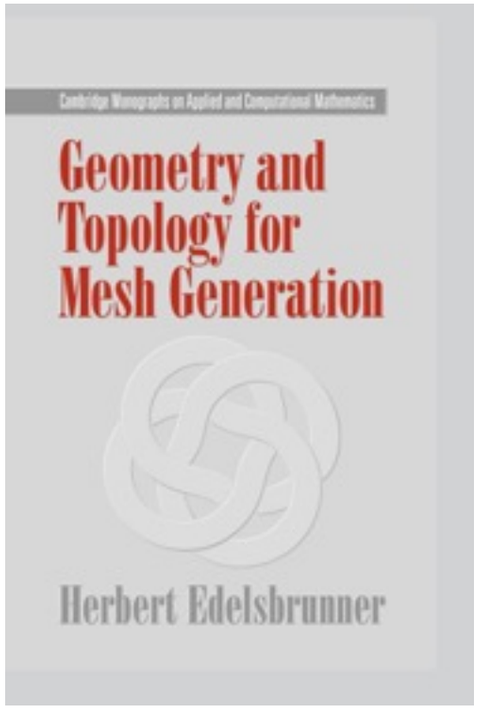
- Mesh research corner, maintained by S. Owen.

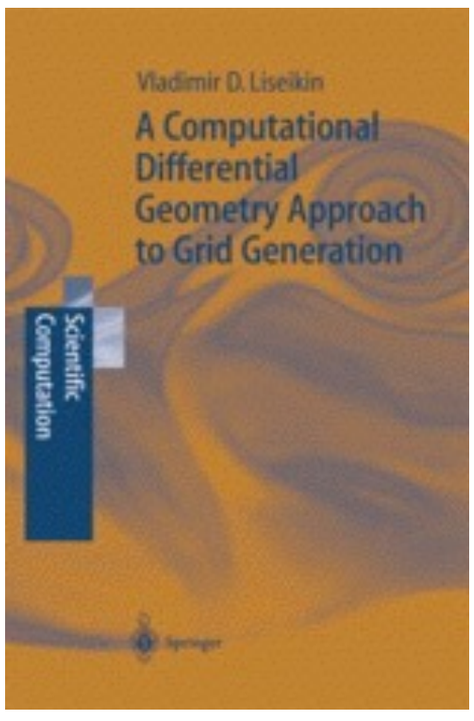
- Mesh generation on the web, maintained by R. Schneider.

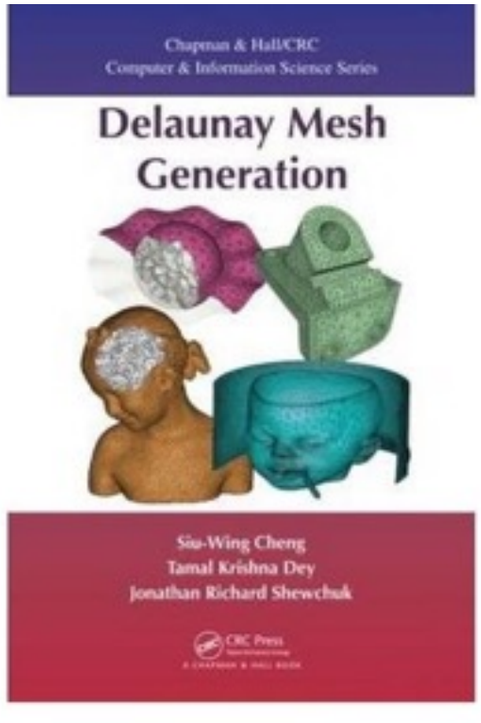
# Literatures, Books

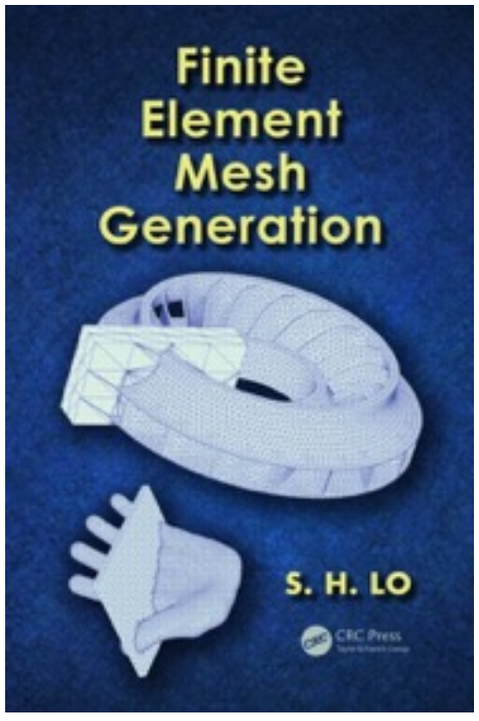


# Conferences



| | Year | Date | Location | Proceedings | Website | Chair |
|---|---|---|---|---|---|---|
| 1 | 1992 | | Northwestern University, IL | | | Ted Blacker, Sandia/Northwestern |
| 2 | 1993 | | Albuquerque, NM | | | Ted Blacker, Sandia/Northwestern |
| 3 | 1994 | October 24-25 | Albuquerque, NM | Proceedings IMR3 | | Ted Blacker, Sandia |
| 4 | 1995 | October 16-17 | Albuquerque, NM | Proceedings IMR4 | | Tim Tautges, Sandia |
| 5 | 1996 | October 10-11 | Pittsburgh, PA | Proceedings IMR5 | | Scott Mitchell, Sandia |
| 6 | 1997 | October 13-15 | Park City, UT | Proceedings IMR6 | | David White, Sandia |
| 7 | 1998 | October 26-28 | Dearborn, MI | Proceedings IMR7 | | Lori Freitag, Argonne National Laboratory |
| 8 | 1999 | October 10-13 | South Lake Tahoe, CA | Proceedings IMR8 | IMR8 | Kenji Shimada, Carnegie Mellon University |
| 9 | 2000 | October 2-5 | New Orleans, CA | Proceedings IMR9 | IMR9 | Steven Owen, Ansys/Sandia |
| 10 | 2001 | October 7-10 | Newport Beach, CA | Proceedings IMR10 | IMR10 | Alla Sheffer, University of Illinois, Urbana |
| 11 | 2002 | September 15-18 | Ithaca, NY | Proceedings IMR11 | IMR11 | Nikos Chrisochoides, College of William and Mary |
| 12 | 2003 | September 14-17 | Santa Fe, NM | Proceedings IMR12 | IMR12 | Jason Shepherd, Sandia |
| 13 | 2004 | September 19-22 | Williamburg, VA | Proceedings IMR13 | IMR13 | Alper Üngör, University of Florida |
| 14 | 2005 | September 11-14 | San Diego, CA | Proceedings IMR14 | IMR14 | Byron Hanks, Sandia |
| 15 | 2006 | September 17-20 | Birmingham, AB | Proceedings IMR15 | IMR15 | Philippe Pebay, Sandia |
| 16 | 2007 | October 14-17 | Seattle, WA | Proceedings IMR16 | IMR16 | David Marcum, Mississippi State University |
| 17 | 2008 | October 12-15 | Pittsburgh, PA | Proceedings IMR17 | IMR17 | Rao Garimella, Los Alamos |
| 18 | 2009 | October 25-28 | Salt Lake City, UT | Proceedings IMR18 | IMR18 | Brett Clark, Sandia |
| 19 | 2010 | October 3-6 | Chattanooga, TN | Proceedings IMR19 | | Suzanne Shontz, Pennsylvania State University |
| 20 | 2011 | October 23-26 | Paris, France | Proceedings IMR20 | IMR20 | William Roshan Quadros, Sandia National Laboratories |

# Commercial Softwares

- Tetmesh-GHS3D, INRIA, France
- MeshSim, SCOPEC, RPI, Simmetrix Inc. USA
- VisTools/Mesh, AreoAstro, MIT, Vki Inc, USA
- GridPro, USA
- GridGen, USA
- …

# Open Source Softwares

- Netgen, TU Vienna, Austra
- Gmsh, Uni. Liege & Uni C. d. Louvain, Belgium
- GRUMMP, Uni. British Columbia, Canada
- Triangle, UC Berkeley, USA
- CGALmesh, INRIA, France
- TetGen, WIAS Berlin, Germany
- …