Scientific Computing WS 2019/2020

Lecture 29

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What was not covered in the course I

The world of partial differential equations is much larger:

- Fluid flow equations: Navier-Stokes, Euler equations
- Elasticity
- Electromagnetic field equations
- Wave equations
- Coupled systems of PDEs

What was not covered in the course II

There are many more discretization methods:

- Higher order finite elements
- Virtual element method: Finite elements on arbitrary polygons
- Discontinuous Galerkin method: allow discontinuities at element boundaries, generalize finite volume fluxes
- Finite volume methods based on triangles or on general polygons
- Multipoint flux finite volume methods
- Galerkin methods in other contexts: Wavelets, Polynomial chaos ...

Linear systems of equations I

- Sparse direct solvers
  - Definition of sparse matrices. Why do we need them ?
  - Sparse matrix computer representation
  - Complexity of LU factorization, LU solve vs. space dimension
  - Advantages and disadvantages of sparse direct solvers

Iterative solution of linear systems I

- Simple iteration scheme
- Convergence criterion
- Precondtioning, examples of preconditioners
- Asymptotic convergence rate vs. spectral radius
- Convergence vs condition number for SPD systems
- Richardson iteration, optimal parameter

Iterative solution of linear systems II

- Diagonal dominant matrices, irreducible matrices
- Perron-Frobenius theorem
- Regular splittings
- M-Matrices and regular splittings
- Convergence of simple preconditioners in the case of non-SPD matrices

Iterative solution of linear systems III

- Krylov subspace methods
- Convergence estimate of the CG method
- Complexity of iterative solvers vs. space dimension

Triangulations

- Admissible triangulations
- Delaunay triangulation of point sets
- Boundary conforming Delaunay triangulations
- Voronoi diagram

Partial differential equations

- Strong formulation of stationary diffusion, reaction-diffusion and convection-diffusion equations
- Transient problems
- Dirichlet, Robin and Neumann boundary conditions
- Sobolev norms
- Weak formulation of diffusion problem with Dirichlet or Robin boundary conditions
- Lax-Milgram theorem for diffusion problem
- Maximum principle for diffusion problem

Finite element method

- Cea's lemma
- Galerkin method
- P1 finite elements for diffusion problem on triangular meshes:
  - Local basis functions
  - Global basis functions
  - Results of convergence estimates, condition number

Voronoi Finite Volume method

- Derivation of the method for diffusion problem
- Resulting matrix properties
- Discrete maximum principle
- Adaptation to convection-diffusion problems, upwinding

Additional topics

- Finite volumes for convection-diffusion problems, upwinding
- Finite volumes for nonlinear problems
- Time dependent problems, implicit vs explicit Euler
- Solution of nonlinear systems of equations
- Paradigms for parallelization: SIMD vs MIMD, Shared vs. Distributed memory

Exams

- Please check the dates
- Please ensure that Annett Gillmeister has the yellow sheets
- Exams will be in MA379
- I offer additonal consultations at Weierstrass institute, please inquire by email