I. Trap model

Random walk on a lattice with each site containing trap corresponding to mean waiting time $\tau$ with

$$\rho_s(t) = \alpha t^{-\alpha} \quad (t \geq 1)$$

and $0 < \alpha < 1$.

Can be applied to variety of physical problems related to disordered media (photoactive solids [1]), capacitance networks [2], and lattice vibrations [3]. Also toy model for spin glasses [4].

Master equation 1D ring

\[ \dot{p}_i = -Lp_i \quad (L : \text{Laplacian}) \]

In components:

\[ \dot{p}_i = \frac{p_{i-1} - 2p_i + p_{i+1}}{2\tau} \]

1. $L$ not symmetric but $L$ and $L^T$ similar.
2. Let $X_j$ be eigenvector of $L$ to eigenvalue $\lambda_j$ and $Z_j$ eigenvector of $L^T$ to same eigenvalue.
   \[ X_j^T Z_i = \delta_{ij} \]
   and thus
   \[ p = \sum_j (p_j^T Z_i) X_j e^{-\lambda_j t} \]

Spectrum

\[ \lambda_1 < \lambda_2 < \ldots < \lambda_N \]

$\lambda_1 = 0$ corresponds to equilibrium state $p_{eq}$.

II. Participation ratio

\[ Y_2(t) = \langle X^{(1)}(t) = X^{(2)}(t) \rangle \]

Population average:

\[ Y_2(t) = \langle p \rangle = \langle p^T p \rangle \]

Same starting position means $p_0^2 = 1$.

Average over $p_0$

\[ \langle Y_2(t) \rangle_{p_0} = \frac{1}{N} \sum_{ij} G_{ij} e^{-(\lambda_i + \lambda_j)t} \]

with $G_{ij} = \langle X_i^T Z_j \rangle \langle X_j^T X_i \rangle$.

III. Dynamical localization

Dynamical localization

\[ \lim_{t \to \infty} \lim_{N \to \infty} \langle Y_2(t) \rangle_{p_0, \tau} > 0 \quad \text{if} \quad D = 1 \]

Radius of gyration $\langle R_G^2 \rangle_\tau$:

- mean squared distance between both walkers

- Radius of gyration: no plateau!

- Participation ratio describes kind of localization other than radius of gyration.

Can dynamical localization be connected to the properties of the Laplacian’s spectrum and eigenfunctions?

IV. Connection to eigenfunctions

Distinguish between influence of eigenvalues and eigenfunctions in

\[ \langle Y_2(t) \rangle_{p_0, \tau} = \sum_{ij} \langle G_{ij} e^{-(\lambda_i + \lambda_j)t} \rangle_\tau \]

Approximation

Neglect the correlations:

\[ \tilde{Y}_2 = \sum_{ij} \langle G_{ij} \rangle_\tau e^{-\lambda_i t} e^{-\lambda_j t} \]

Further $\tilde{Y}_2(G^{(1D)}, \lambda^{(1D)})$: Mix $\langle G \rangle_\tau$ of 1D with $\lambda$ of 3D.

Figure top right: Temporal evolution of the approximation $\tilde{Y}_2$ in the 3D and 3D case as well as $Y_2(G^{(3D)}, \lambda^{(3D)})$ compared to the exact $\langle Y_2 \rangle_{p_0, \tau}$ (1D). Disorder average over 100 landscapes with 1331 sites each. drawer figures right: The first $10 \times 10$ entries of $\langle G \rangle_\tau$ for 1D and 3D. Grey colored columns $\langle G_{ij} \rangle_\tau$ positive, white: sign negative.

Contribution of summands

Take the eigenvalues only from 3D spectrum, $G$ from respective dimension.

References